APPLIED MECHANICS

Reviews

A CRITICAL REVIEW OF THE WORLD LITERATURE IN APPLIED MECHANICS AND RELATED ENGINEERING SCIENCE

A Review of Unsteady Aerodynamics of Potential Flows, I. E. Garrick, 89

APPLIED MECHANICS

Reviews

A REVIEW OF UNSTEADY AERODYNAMICS OF POTENTIAL FLOWS

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SCOPE AND SIGNIFICANCE

THIS review, dealing with the addition of dimension "time" to steady aerodynamics, is limited chiefly to theoretical nonsteady wing characteristics, a subject of importance in the field of aeroelasticity as in aircraft flutter, in dynamic stability and maneuverability, gusts, transient, pulsating, and buffeting flows. Of significance in the dynamics of these problems are the roles of the aerodynamic inertia, damping, and restoring forces and moments, and the time lag behavior between the motion and the air forces. These response characteristics may be required for a variety of components, configurations, vibration modes, attitudes, and ranges of altitude, speed, and frequency. Great gaps naturally exist in our knowledge of the subject even for idealized potential flows; nevertheless, important information exists in many scattered papers. Reference (1)[1], for example, refers to over 160 papers; references (2, 3, 4, 5) list numerous other sources.

ACTIVITY AND GENERAL LITERATURE

The subject was effectively launched in 1923 by W. Birnbaum, a short-lived student of Prandtl. His approach (two-dimensional, small perturbations of an incompressible fluid) made use of the concepts of harmonically oscillating vorticity distributions bound over the airfoil and free floating in the wake, the total circulation being zero by Kelvin's theorem. The Kutta condition for smooth flow at the trailing edge was invoked implicitly and the air forces expanded in powers of the frequency ω, including terms in $\log \omega$. Another basic approach utilizing conformal mapping and emphasizing the transient flow cases of a sudden angle of attack (unit step) and accelerated rectilinear motion was made by H. Wagner in 1925. There followed international activity: in England, by Glauert in 1929, the case of the accelerated and oscillating pitching airfoil, and by Duncan and Collar in 1932, the damped exponential case; in the United States by Theodorsen in 1934, the explicit expressions for the air forces involving Bessel or Hankel functions for the harmonic oscillations of an airfoil, and by Garrick in 1936 the drag and propulsion of an oscillating airfoil, related to the immemorial problem of bird flight. In Italy, Cicala in 1935 independently obtained Theodorsen's results by novel and useful methods, later in 1937 used in three-dimensional problems. Küssner, in the 1929 German DVL yearbook, followed up on Birnbaum's series expansion methods, and in 1936 in *Luftfahrt-*

forschung pointed to the Laplace transformation relations existing between the transient unit step (Wagner) function and the harmonically oscillating (Theodorsen) response. These superposition methods find applications in gust work and can lead to results for arbitrary time dependence. A timely exposition, in which a gust penetration function (of Küssner) was first correctly obtained, was given in 1938 by von Kármán and Sears. They introduced a concept of quasi-steady flow, to which nonsteady results can be usefully referred, for which the instantaneous normal velocity distribution of the airfoil is frozen in time.

Basic studies for the subsonic and supersonic nonsteady flow of a compressible fluid were made in 1937 and 1938 by the young Italian, C. Possio, who died in 1945 at 31 years of age. He utilized singularities of the wave equation corresponding to moving acoustic sources and doublets and made effective use of the acceleration (pressure) potential methods revived by Prandtl in 1936. The unique issue of *Luftfahrtforschung* of December 10, 1940, devoted entirely to nonsteady flows papers *inter alia* by Küssner, Schwarz, and Söhngen) was the forerunner in Germany of further studies during wartime. Recent monographs (2, 3) contain well over 100 German references alone. Professor G. Temple has submitted for publication a chapter on unsteady flow for Vol. III of "Modern Developments in Fluid Dynamics." Many contributions are emanating from the research laboratories, NACA (USA), RAE and NPL (England), NLL (Holland), ONERA (France), and others. A book by Nekrasov (6) summarizes the Russian scene, however, only up to 1942: additional papers have appeared in the Russian journal, *Appl. Math. Mech.* (*Prikl. Mat. Mekh.*).

PROBLEMS AND METHODS

The governing partial differential equation in its general nonlinear form has been shown to be given by a form of the wave equation in which the velocity of propagation is the local velocity of sound. It thus contains all the difficulties inherent in discussions of waves of finite amplitude and formation of shocks, topics of current interest that underlie higher order treatments but are not within the scope of this review. The chief material extant deals with small perturbations which lead to the standard linearized equation of wave motion and which reduce the problem "almost" to mathematics. The mathematical formulation involves boundary conditions, also usually linearized and of mixed type involving the potential and its derivatives, plus phenomenological conditions like the Kutta condition for subsonic flow, conditions for attached shocks in supersonic flow.

[1] Numbers in parentheses refer to Bibliography at end of the paper.

The formulation as a general integral equation in terms of the acceleration potential was dealt with by Küssner (1940) and is given in terms of the velocity potential in a paper containing many references by Reissner (AMR 4, Rev. 523). The Possio integral equation (two-dimensional subsonic case) has been the subject of much inquiry, but an explicit solution such as exists in the incompressible flow case has not been found. Approximate methods are: collocation (Possio, Frazer); iteration, making use of the known result for the incompressible flow (Dietze); kernel expansion methods (Schade, Fettis). There is also a classical boundary-value formulation, an expansion in series of orthogonal functions; the subsonic flow problem leads to the use of Mathieu-Hankel functions (Reissner-Sherman, Timman, Haskind, AMR 4, Revs. 3617, 2142; 1, 158; also A. E. Billington, *Australian Aero. Res. Rept.* A65, 1949). Significant numerical results by this method for the plane case have been recently reported (7).

That steady transonic flow, even for small perturbations, is essentially nonlinear is well known, but that this nonlinearity does not necessarily extend to the range of high frequencies was shown by order of magnitude studies of the differential equations by Lin, Tsien, and Reissner (AMR 3, Rev. 120). Contributions for near sonic transient flows were made by Heaslet, Lomax, and Spreiter (AMR 2, Rev. 1420); for the oscillatory case by Rott (AMR 4, Rev. 788), and in more numerical detail by Nelson and Berman (8); for accelerated motion by Biot and by Gardner and Ludloff (AMR 3, Revs. 311, 2399). The effect on the small perturbations of a strongly disturbed underlying flow has been touched on by Davies (AMR 3, Rev. 2010) and shown negligible for weak attached shocks by Sewell (AMR 4, Rev. 1678).

Although the subject is only a few years old, there looms the promise of considerable activity on problems of nonsteady flow past wings at supersonic speed. There are presented a multiplicity of combinations of conditions and planforms: simple (purely supersonic, Garrick and Rubinow, AMR 1, Rev. 343) and not simple, with supersonic and subsonic leading or trailing edges. Evvard's upwash methods have assisted solution of some low-frequency time-dependent and some transient problems (AMR 4, Rev. 3952; see also Krasilshchikova, AMR 3, Rev. 537); the rectangular wing has been treated by various methods by Miles, Stewart and Li, Chang, and Watkins; the (narrow) delta wing by Watkins (9) by expansion in powers of the frequency, and by Stewartson (AMR 4, Rev. 2121) by Laplace transform methods. (See also Cal. Tech. thesis 1951 by Hipsch.) A second-order solution for thickness and angle of attack has been given by Wylly (Cal. Tech. thesis 1951) for the two-dimensional oscillating airfoil. The nonsteady slender-body problem has been considered by Stewartson, and by Miles (AMR 4, Rev. 3003).

A large void exists in the three-dimensional nonsteady theory for subsonic compressible flow. Considerable effort has gone into the incompressible-flow wing theory but the results are far from satisfying, in exactness or simplicity. The approximations (Cicala, Küssner, Reissner, W. P. Jones, et al) lead to line or strip analysis rather than true surface theory. The work of R. T. Jones in 1939, based on physical reasoning and on the interrelations between transient and oscillatory flow, has the advantage of simplicity but is not easily assessed or improved. The translation and rotation of an oscillating wing of circular planform have been treated approximately by Krienes and Schade (AMR 1, Rev. 538) and by Kochin (10) by the inverse method of superposition of modes associated with some chosen standard loadings, however, without consistency in agreement of results. Sweepback has been treated by two-dimensional strips and effective normal velocity components (AMR 4, Rev. 1309); also, extending Reissner's finite span work by Turner (AMR 3, Rev. 2741). The low-aspect-ratio wing was considered by Garrick and Reissner (1) and by Ashley (MIT D.Sc. thesis 1951) as direct extension of the concepts of R. T. Jones (AMR 2, Rev. 902) for steady flow.

The useful application of the quasi-steady approach, for example, in the study of dynamic stability of tailless designs, by expansion in first-order terms in frequency, including log terms if necessary, has been emphasized recently by Miles, Goland, Statler, et al. This points up the fact that any improvements in methods and results for steady-flow wing theory can be directly adapted to unsteady flow. The potential usefulness of high-speed computing machines in alleviating the numerical effort required is great.

REMARKS ON NONPOTENTIAL FLOWS

Some brief remarks supplementing the foregoing reviews on some practical points are appropriate. An application may fail on mathematical or on physical grounds; for example, the application to the unsteady moments of control surfaces suffers from both the mathematical idealization (mean line concept) and because of large boundary-layer effects. Viscous effects, flow separation, and the boundary layer are important in many unsteady flow problems but are relatively obscure. It is therefore essential that theory and experiment work hand in hand. The experimental methods have the threefold objective: Checking the theory and thereby establishing techniques; establishing deviations from and modifications to the theory; and furnishing information on situations not readily amenable to theory. The techniques and methods are difficult and still in an evolutionary state; for example, in wind tunnels, flow roughness and unsteady wall interference (11, 12) can be important. The techniques vary from indirect, as in flutter research, to measurement of response and reactions in controlled free and forced oscillations, and more directly to force and pressure measurements. However, this side of the subject is a separate story and one that can be of importance even in nonaeronautical fields, as attested to by the Tacoma bridge failure in 1940.

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3 Küssner, H. G., and Billing, H., *Hydro- and Aerodynamics* Editor, A. Betz, 141–198, 1950. *CADO Transl. ATI* 72854
4 Williams, J., Aircraft flutter, AMR 4, Rev. 4264.
5 Greidanus, J. H., A review of aeroelasticity. AMR 4, March 1951, feature article.
6 Nekrasov, A. I., Wing theory for nonstationary flow, Moscow *Acad. of Sci. USSR*, 1947.
7 Timman, R., van de Vooren, A. I., and Greidanus, J. H., Aerodynamic coefficients of an oscillating airfoil in two-dimensional subsonic flow, *J. aero. Sci.* 18, 797–802, 1951.
8 Nelson, H. C., and Berman, J. H., Calculations on the forces and moments for an oscillating wing-aileron combination in two dimensional potential flow at sonic speed, *NACA TN* 2590, 1951.
9 Watkins, C. E., Air forces and moments on triangular and related wings with subsonic leading edges oscillating in supersonic potential flow, AMR 5, Rev. 481.
10 Kochin, N. E., *Prikl. Mat. Mekh.*: 4, 3-32, 1940; 6, 287-316, 1942.
11 Watkins, C. E., and Runyan, Harry L., Considerations of the effect of wind-tunnel walls on oscillating air forces for two-dimensional subsonic compressible flow, *NACA TN* 2552, 1951.
12 Timman, R., The aerodynamic forces on an oscillating aerofoil between two parallel walls, Rev. 766 in this issue.

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AMR 2, Revs. 94, 96, 221, 244, 245, 324, 333, 346, 350, 786, 1053, 1160, 1313, 1425, 1529.
AMR 3, Revs. 98, 113, 123, 143, 150-154, 540, 541, 563, 757, 909, 1120, 1135, 1308, 1541, 1562, 1747, 1991, 2000, 2031, 2400, 2724, 2754.

AMR 4, Revs. 284, 288, 315, 324, 329, 330, 332, 333, 363, 379, 380, 787 790, 792, 795, 832, 841, 847, 1249, 1257, 1267, 1310, 1311, 1706, 1737, 1739, 2094, 2118, 2128, 2142, 2143, 2224, 2588, 2591, 3004, 3007, 3122, 3282, 3306, 3596, 3600, 3626, 3628, 3631.

Theoretical and Experimental Methods

(See also Revs. 599, 624, 631, 633, 645, 659, 715, 812, 815, 848, 858, 859, 871, 915)

572. **Cowling, T. G., A new method of numerial integration of the equations of the laminar boundary layer,** *Aero. Res. Counc. Lond. Rep. Mem.* 2575, 14 pp., May 1945, published 1951.

Method is applicable to incompressible flow. Velocity profiles are calculated at successive sections across the boundary layer. At each section trial values of velocity are assumed and space derivatives found by finite difference methods. Residuals, interpreted as time derivatives of the velocity, are found from the differential equations. Velocities are then adjusted to make residuals vanish. Method is somewhat analogous to that of Southwell.

Method is applied to two problems which were solved by Hartree on the differential analyzer [AMR **3**, Revs. 1343, 1546]. Author says results are more accurate and more easily obtained than by Hartree's method. Method of extrapolation for finding point of separation is also given.

Reviewer notes that the last term of author's principal equation (Eq. 6) should read $\partial^2 u/\partial \eta^2$. R. C. Roberts, USA

573. **Ramsayer, K., Function calculators with first- and higher-order interpolation** (in German), *Z A M M* **31**, 10, 301–309, Oct. 1951.

Paper describes application of mechanisms for storing function values to ordinary calculating machines. If a linear interpolation is to be used, either function values only or also values of the differences need to be stored. A simple mechanical construction to that end is described, and the necessary manipulations are discussed. The accuracy that can be obtained is investigated and illustrated on simple functions like sin x, etc. In case of higher-order interpolation, the function may be represented in each interval by a power series, the coefficients of which have to be stored. The derivation of these power series and the obtainable accuracy is given. The total number of data to be stored are shown by examples to be reduced considerably in this way, but more manipulations are required. A. van Wijngaarden, Holland

574. **van Wijngaarden, A., Decimal-binary conversion and deconversion,** *Math. Centre, Amsterdam, Comput. Dept. Rep.* 130, 11 pp., 1951.

Report contains an extensive table (with explanation) for converting from base 10 to base 2 (and conversely) which will be of use to institutions with digital computing machines operating in the base 2. Conversion (and deconversion) of $5n$ decimal digit numbers can easily be made with n table look-ups.
C. L. Perry, USA

575. **Robinson, J.-R., and Courbon, J., Numerical calculations by machine in civil engineering** (in French), *Ann. Ponts Chauss.* 121, 4, 421–462, July-Aug. 1951.

Compilation of rules and formulas of numerical analysis, especially interpolation and integration with equal intervals, with instructions for the use of calculating machines. Author prefers electrical desk machines without automatic multiplication.
F. A. Willers, Germany

576. **Esnault-Pelterie, R., Dimensional analysis and metrology,** F. Rouge et Cie, S. A., Lausanne, Aug. 1950, xiv + 112 pp.

In 1948, author published "L'Analyse Dimensionelle" [F. Rouge et Cie, Lausanne, June 1948, 236 pp., and subsequently prepared an English version ["Dimensional Analysis," same publisher, 1950, xvi + 270 pp] wherein "many persisting imperfections (in the French edition) have been rectified and appreciable developments have been introduced." English edition closes with the statement: "Although this English edition has been entirely recast from the French, it is certain that many imperfections still escape the author's vigilance. He will feel much indebted to those who will draw his attention to the remaining inaccuracies." In response, interested readers "presented some observations which proved fruitful and certain remaining imperfections could then be amended." Again, "it appears desirable that the English publication be put into harmony with the French edition." Thirdly, certain points relative to the mks system of units were incompatible with some of the author's work on dimensional analysis. Accordingly, "these different reasons induced me to consider publishing a complement to that quite recent English edition."

Present work has two parts; the first five chapters comprise "development of a number of items which appear neither in the French nor the English edition. Paragraphs therein follow the completed text of the English edition." The second part comprises chapter Vb devoted to discussion of the mks system of units from the standpoint of the author's work on dimensional analysis. In particular, author's contention that attribution of four principal units to the mks system is incorrect, as one can be derived from the other three, is discussed thoroughly.

Reviewer found much of interest in a careful study of author's three books. However, since the English edition "is not simply a Treatise on Dimensional Analysis, but also constitutes an attempt to take the opportunity offered by this subject of deep significance to discriminate between different notions which are proved to be constantly intermingled, even by the best authors," the considerable amount of interwoven epistemological and philosophical material makes it rather difficult reading. This can be somewhat ameliorated by prefacing perusal of author's books by a preliminary reading of Langhaar's recent excellent book on dimensional analysis [see AMR **4**, Rev. 4042].
Thomas J. Higgins, USA

577. **Somers, E. V., and Cyphers, J. A., Analysis of errors in measuring thermal conductivity of insulating materials,** *Rev. sci. Instrum.* 22, 8, 583–586, Aug. 1951.

578. **Rosenberg, R. M., and Wang, A. J., Periodic solutions of a nonlinear differential equation,** *Univ. Wash. Engng. Exp. Sta. Seattle Bull.* no. 118, Aero. Ser. no. 1, 77–87, 1951.

Paper considers second-order, ordinary differential equation, linear (with constant coefficients) except for term proportional to square of first derivative, without a linear first-derivative term, and with a periodic forcing function. Corresponding equation without forcing function has been treated by Milne [*Univ. Oregon Publ., Math. Series*, 1, 1, no. 1, Mar. 1929], who has tabulated solutions of a normalized form for initial velocity as parameter. Authors approximate forcing function by a stepwise-varying function, whose constancy in any step permits reduction of differential equation to Milne's normalized form. Method of solution consists in guessing initial transient values of phase coordinates and using Milne's tables to continue solution to any step. Steady-state solution is obtained when continuation of process yields a closed locus of phase coordinates. Comparison of technique with method of "equivalent viscous damping" is made for a number of examples; close agreement in results of the two methods is found. J. J. Gilvarry, USA

579. Abramowitz, M., Table of the integral $_0\int^x e^{-u^3}\,du$, *J. Math. Phys.* **30**, 3, 162–163, Oct. 1951.

580. Abramowitz, M., Tables of the functions $_0\int^\phi \sin^{1/3}x\,dx$ and $(4/3)\sin^{-1/3}{}_0\int^\phi \sin^{1/3}x\,dx$, *J. Res. nat. Bur. Stands.* **47**, 4, 288–290, Oct. 1951.

581. Segre, B., On the rational solutions of homogeneous cubic equations in four variables, *Math. Notae, Bol. Inst. Mat., Rosario, Arg.* 11, 1–2, 1–67, 1951.

An algebraic curve or surface is rational in case it is representable by one or a system of equations $f_r(x_1, x_2, x_3, x_4) = 0$, where each f_r is a polynomial with rational coefficients. A point with homogeneous coordinates (x_1, x_2, x_3, x_4) is rational in case the ratios of the coordinates are rational. Author considers the existence of rational points on a rational cubic surface. A typical theorem is that "any rational cubic surface contains either no rational points or an infinity of rational points."

A. S. Householder, USA

582. Tranter, C. J., Integral transforms in mathematical physics, New York, John Wiley & Sons, Inc.; London, Methuen & Co., Aug. 1951, ix + 118 pp. $1.50.

Pocket-size monograph dealing briefly with Laplace, Fourier, Hankel, and Melvin transforms. Treatment is not elementary— considerable mathematical maturity and computational facility on the part of the reader is assumed (contour integration, etc.)— and several delicate questions (commutability of operations, integral convergence, etc.) are largely by-passed. Exposition is largely by example and computational exercise, with little effort to develop insight or "feeling" for the subject either as a mathematical discipline or practical tool. Reviewer feels book is suitable as reference or refresher but not as text for first study. More extensive tables of transforms would have increased its usefulness. Good bibliography and index, well set up and well printed.

L. B. Hedge, USA

583. Filippov, A. P., The deformation of an elliptic plate with simply supported boundary under the action of concentrated loads (in Russian), *Inzhener. Sbornik* 5, 2, 71–82, 1949.

Analytically, the boundary-value problem considered in this paper consists in the determination of a real-valued function $w(x, y)$ (deflection of a plate) defined on an ellipse in the (x, y)-plane, such that: (a) In the interior of the ellipse w satisfies, save at a finite number of points of the interior (where concentrated loads normal to the plane of the plate are present), the partial differential equation

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right) = \frac{1}{D}\,f(x, y)$$

where $f(x, y)$ is a given function (the distributed load normal to the plane of the elliptic plate) and D is an elastic constant; at each of the exceptional set of points w behaves like $Cr^2\log r$ plus a regular function, where C is a given real number (proportional to the intensity of the given concentrated load at the point) and r is the Euclidean distance from the point. (b) On the elliptical boundary w satisfies the conditions (simply supported plate) $w = 0$,

$$M = -Dv\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right)$$
$$-D(1-v)\left(\cos^2\theta\,\frac{\partial^2 w}{\partial x^2} + \sin^2\theta\,\frac{\partial^2 w}{\partial y^2} + \sin 2\theta\,\frac{\partial^2 w}{\partial x\partial y}\right) = 0$$

where D and v are elastic constants, and θ is the angle between the outer normal to the boundary and the positive x axis. The boundary-value problem is first formulated in terms of elliptic coordinates. A formal solution is then given, expressed as an infinite series whose coefficients may be determined numerically by solving linear equations, in the special case of two concentrated loads, of equal magnitude and direction, symmetrically placed with respect to the major axis of the ellipse. The particular case of a concentrated load at the center of the plate is dealt with numerically.

Courtesy of Mathematical Reviews J. B. Diaz, USA

584. Schwank, F., Boundary-value problems for physicists, mathematicians and engineers [Randwertprobleme für Physiker, Mathematiker und Ingenieure], Leipzig, B. G. Teubner Verlagsges., 1951, vi + 406 pp., 147 figs. $6.67.

Book is written as a quick and easy introduction to the boundary-value problems of mathematical physics. The object is to survey typical boundary-value problems so that the reader who is interested in any particular phase may then consult more extensive treatises on the subject.

Starting with the simplest boundary-value problem of a mass vibrating at the end of a spring, author advances into the more complicated problems of the vibrating string and membrane. Early in the book the problem of the vibrating string is set up, first, as a problem in partial differential equations, then as a problem in integral equations, and, finally, as a problem in the calculus of variations.

In order to elaborate on these various methods, author includes chapters on complex function theory, potential theory, integral equations, partial differential equations, and the calculus of variations. Bessel functions are introduced in connection with the problem of the vibrating membrane with a circular boundary. Biharmonic functions are introduced in connection with the deformation of plates. Sections on Legendre polynomials, the Ritz method in the calculus of variations, the general wave equation and the heat equation are also included.

Reviewer finds the book written simply and clearly and believes volume should be favorably received by people interested in these types of problems. A. Devinatz, USA

585. Estrin, T. A., and Higgins, T. J., The solution of boundary-value problems by multiple Laplace transformations, *J. Franklin Inst.* **252**, 2, 153–167, Aug. 1951.

The necessary theory for the solution of partial differential equations by multiple Laplace transforms is given. The theory is followed by its application to two boundary-value problems, one in electrostatics, the other in heat conditions.

A. W. Wortham, USA

586. Fil'chakov, P. F., Method of successive mapping of slits (in Russian), *Dokladi Akad. Nauk SSSR (N.S.)* 78, 3, 413–416, May 1951.

Author examines the complex-function transformation of a boundary which, in its original form, includes a length of the horizontal axis and two vertical slits extending downward from the axis, into a single straight line segment. The first mapping eliminates one of the slits and transforms the other into a curved line. Approximating this curved line by a straight line, or a circular arc, leads to various second mapping functions which complete the transformation. Errors are estimated and the method generalized to more than two slits. Over a large, useful range of geometric parameters the errors of the approximations are small. Author makes specific reference to the problem of the flow of water in a permeable material beneath a dam.

Richard E. Kronauer, USA

587. Otsuka, S., **Solution of wing-lattice problem by means of electric method** (in Japanese), *Rep. Inst. Sci. Technol. Tokyo*, 4, 1/2, 49–55, 1950.

The region outside a wing lattice in the ζ-plane is transformed into a region D in the ζ_2-plane by means of known functions $\tanh (\pi\zeta/a) = \zeta_1$, $b\zeta_1 + c = {}^1/_2(\zeta_2 + 1/\zeta_2)$. A thin steel plate is cut in the shape of D. An electric current is sent between two points A, B, which correspond to $\zeta = \infty$, and the electric potential on the circumference is measured. Hence, velocity distribution on the wing lattice can be easily calculated by numerical or graphical differentiation. The method is carried out for an example, obtaining satisfactory result.   Isao Imai, Japan

588. Ōtsuka, S., **The influence of the size of electrode in the electric method of solution of wing-lattice problem** (in Japanese), *Rep. Inst. Sci. Technol. Tokyo*, 4, 5/6, 150–154, 1950.

The effect of finite dimension of the electrodes A, B in the author's electric method (see preceding review) is investigated analytically by taking the simple case that a circular electrode of radius R is placed at a distance l from a straight-line boundary. It is found that the relative error in the measured potential is at most $(R/l)^2$.   Isao Imai, Japan

589. Seth, B. R., **Boundary conditions interpreted as conformal transformation**, *Proc. Amer. math. Soc.* 2, 1, 1–4, Feb. 1951.

The author makes the obvious remark that many boundary-value problems to which function theory is applicable can be formulated as problems in conformal mapping, and demonstrates for certain general types of boundary conditions, where polygonal boundaries are involved, that the problems can be solved (with help of Schwarz-Christoffel) in essentially closed form. For curved boundaries the method is reduced to that for polygonal boundaries by approximation.
Courtesy of Mathematical Reviews   D. Gilbarg, USA

590. Kertz, W., **Theory of tidal-shaped air vibrations as eigenvalue problem** (in German), *Ann. Meteor. Beiheft*, 31 pp., 1951.

The equations of motion in an autobarotropic atmosphere on an earth with surface irregularities are formulated and expressed, by means of operators, in terms of a linear eigenvalue problem. Starting from the solutions for a nonrotating earth with smooth surface, the theory of perturbations is applied to find the deviations produced by the earth's rotation and by an obstacle corresponding to the cordilleras in western America. The comparison of observational results on tidal atmospheric oscillations with theory shows qualitative agreement.

From author's summary

591. Anonymous, **List of current papers published by the Aero. Res. Counc., nos. 1–50**, *Aero. Res. Counc. Lond. curr. Pap.* 50, 5 pp., July 1951.

592. Leitner, A., and Spence, R. D., **The oblate spheroidal wave functions**, *J. Franklin Inst.* 249, 4, 299–321, Apr. 1950.

The following quotations are from the introduction: "While a number of authors have discussed aspects of the [oblate spheroidal wave] functions, we feel that a relatively comprehensive analysis of these functions, which now command wide interest, is lacking"; and "The viewpoint adopted here, in the analysis of the functions at small arguments, was previously used by Page [*Phys. Rev.* (2) **65**, 98–110, 111–117, 1944]. In this discussion of the prolate spheroidal functions, simple power series are used. This method is of advantage, in particular for the radial func-

tions, since the series of spherical Bessel functions converge slowly at small arguments [in fact, they diverge if $|\xi| < 1$] and do not explicitly show the properties of these functions near the singular points of the differential equations"; and "While not all the material presented is original, it is hoped that it will serve to unify the subject by pointing out the interrelations among the approaches of the various authors in the field." The paper contains expressions for the first five coefficients of a power series for the eigenvalues of spheroidal wave functions. [Simpler expressions have been given by the reviewer.] As to the authors' statement, "Since the spherical Neumann functions possess a singularity at the origin neither (73), (74), nor (75) and (76) converge well unless $\epsilon\xi$ is considerably greater than unity," reviewer observes that any of these series diverges inside the unit circle of the complex ξ plane. Typical wave functions representing spherical waves and plane waves are expanded in spheroidal wave functions. Most important are the authors' tables and graphs. The eigenvalues α_{lm} of $d/d\eta$ $[(1 - \eta^2)du_{lm}/d\eta] - [m^2/(1 - \eta^2) - \alpha_{lm} + \epsilon^2(1 - \eta^2)]u_{lm} = 0$ are given for $m = 1$, $l = 4(1)8$; $m = 2$, $l = 2$, $l = 4(1)9$; $m = 3$, $l = 4(1)10$; $m = 4$, $l = 4(1)11$ as a function of ϵ with $\epsilon = 1(1)5$. The number of significant figures varies from 5 to 9. Apart from their graphs the eigenfunctions $u_{lm}(\eta)$ are tabulated for $\eta = 0(0.1)1$, $\epsilon = 1(1)5$; $m = 0$, $l = 0(1)5$; $m = 1$, $l = 1(1)6$; $m = 2$, $l = 2$, 3; (4D). The norms of most of these functions, as well as the values of certain constants associated with the radial functions, are listed also.
Courtesy of Mathematical Reviews   C. J. Bouwkamp

Mechanics (Dynamics, Statics, Kinematics)

(See also Revs. 601, 603, 936)

593. Steward, G. C., **On the cardinal points in plane kinematics**, *Phil. Trans. roy. Soc. Lond.* (A) 244, 875, 19–46, Sept. 1951.

Points are the same as "poles" discussed by Bereis [AMR 4, Rev. 4376] three months earlier. Both papers cover much the same ground but with differences which make both worth studying. Steward assumes more mathematical knowledge on the part of the reader and makes less reference to design applications. In reviewer's opinion, Steward establishes more concisely a clear picture of the basic significance of these points.

A. S. Hall, USA

594. Artobolevskii, I. I., **Two mechanisms for drawing of curves of higher order** (in Russian), *Dokladi Akad. Nauk SSSR (N.S.)* **80**, 5, 717–719, Oct. 1951.

Given a diameter AK of a circle, and the tangent at K, a variable secant through A cuts the circle in C and the tangent in N. Then, if G is a point on the secant so that $AG = CN$, the locus of G is the cissoid of Diocles. This definition is mechanized by pivoting a diametral link CBD at the center B and letting D drive a parallelogram linkage $BDFM$ where the fixed line BM is perpendicular to AK. The crosshead at G is the intersection of DF and AC. [See also AMR 5, Rev. 335.]

If NP is parallel to AK and CP is perpendicular to AK, then the locus of P is the witch of Agnesi. This definition is mechanized with slides, right-angled bars and crossheads. [See "Curves," R. C. Yates, 1946, p. 28 and p. 229.]

Michael Goldberg, USA

595. Binark, H., and Erbakan, N., **Investigation of the change of track due to wheel-suspension of motor cars** (in German), *Bull. tech. Univ. Istanbul* 3, 1, 22–44, 1950.

Problem is presented under simple geometrical constructions. Authors record different kinds of suspensions and include a graph

about a particular case of track change. Comparison is made between actual devices with corresponding schemes. Then the dynamical aspect is considered. Authors develop classical Euler's equations for polar movement. Gyroscopical effect of change of track is presented and solved by those equations. Article closes with a summarizing graph.

Two conclusions are deduced: Well-dimensioned elastic shafts are better than rigid shafts from the standpoint of track and height changes. Pendular shafts impose longer arms to prevent changes considered from becoming important. From this standpoint, pendular shafts are worse than elastic and rigid shafts.

Reviewer finds article to be of didactical importance and mathematically correct. Jorge Carrizo Rueda, Argentina

596. Aizerman, M. A., A sufficient condition for the stability of a class of dynamic systems with variable parameters (in Russian), *Prikl. Mat. Mekh.* **15**, 3, 382–384, May-June 1951.

For the equation $\ddot{x} + a\dot{x} + [1 + q \cos (2t/\mu)]x = 0$, which occurs in theory of sound and electrical machines, author gives a sufficient condition for $x(t) \rightarrow 0$ as $t \rightarrow \infty$, for all values of μ, $x(t = 0)$, $\dot{x}(t = 0)$. Lyapunov's method is used. A curve of q_{max} vs. a is given. Y. C. Fung, USA

597. Gol'din, A. M., On a criterion of Lyapunov (in Russian), *Prikl. Mat. Mekh.* **15**, 3, 379–381, May-June 1951.

Author elaborates Joukowsky's proof of Lyapunov's criterion of boundedness of solutions of the equation $y'' + p(x) y = 0$ . . (1), $p(x)$ being a positive, bounded, periodic function with period ω. Lyapunov's criterion is $\omega_0 \int^\omega p(x)dx \leq 4$ . . (2). Joukowsky shows that (2) amounts to the requirement that two neighboring roots x_1 and x_2 of an arbitrary solution of (1) be separated by an interval $\mu > \omega$. Present author follows Joukowsky by considering a half wave of a solution of (1) and by setting $x_2 - x_1 = \mu$: $y'(x_1) = m_1$; $y'(x_2) = -m_2$. Integration of (1) between x_1, x_2 yields $_{x_1}\int^{x_2} p(x)y(x)dx = m_1 + m_2$. From this point, through a series of algebraic transformations which cannot be abstracted here, he shows that $\mu_{x_1}\int^{x_2} p(x)dx > 4 + 4/\mu^2 P_{max}$, P_{max} being the maximum of $p(x)$ in (x_1, x_2). If $\mu \leq \omega$, by strengthening the preceding inequality, one gets $\omega_0 \int^\omega p(x)dx = Q(x) > 4 + 4/\omega^2 P_{max} = R(x)$. Hence, $Q(x) \leq R(x)$ is a sufficient condition for $\mu > \omega$ and, therefore, for the boundedness of solutions of (1).

N. Minorsky, France

598. Sakharnikov, N. A., Qualitative picture of the behavior of trajectories near the boundary of the stability region containing a singular point of the center form type (in Russian), *Prikl. Mat. Mekh.* **15**, 3, 349–354, May-June 1951.

Author considers a system A of two differential equations of the first order in terms of x and y as dependent, and t as independent variables, not containing t explicitly. Calling the center, focus, node, and col as singular points of the first kind (s.p.f.k. for short), he formulates the following theorem: If A is a s.p.f.k. of the type center, and if no other s.p.f.k. exist at a finite distance, then the following three cases are possible: (1) Through any point of the (x,y) plane passes one closed trajectory surrounding A. If not all trajectories surrounding A are closed, then the trajectory L, limiting the simply connected domain of closed trajectories surrounding A, either (2a) passes through the point at infinity, or (2b) passes through a col without approaching any other s.p.f.k. at a finite distance. He proves the theorems (1), (2a), and (2b) and indicates examples of such differential equations.

N. Minorsky, France

599. Antosiewicz, H. A., A note on asymptotic stability, *Quart. appl. Math.* **9**, 3, 317–319, Oct. 1951.

Paper gives stability criterion for vector differential equation $dx/dt = A(t)x$; elements of $A = (a_{ij}(t))$, $i, j = 1, 2, \ldots n$, real continuous and uniformly bounded for $t \geqslant t_0 \geqslant 0$; holding also if $\int^\infty [\text{trace } A(t)]dt$ diverges (as occurs when all elements a_{ij} are constants). J. H. Greidanus, Holland

600. Kapitsa, P. L., Pendulum with vibrating suspension (in Russian), *Usp. Fiz. Nauk* **64**, 1, 7–20, May 1951.

A simple pendulum whose point of suspension is in sinusoidal vertical motion is considered. This is a case of parametric excitation described by a Mathieu equation. (Author does not seem to be aware of existing literature on the simple or double pendulum excited in this fashion.) Some approximate quantitative results are derived, but author's main attention is on the details of a physical mechanization of the system as a demonstration experiment. Some results: The existence of a stable equilibrium for an inverted pendulum: parametric resonance at certain excitation frequencies. Interpreting the pendulum as a clock, one finds that if the suspension oscillation frequency is below the natural frequency, the clock gains. From this gain, one can calculate the mean energy of the excitation spectrum.

Robert E. Roberson, USA

Gyroscopics, Governors, Servos

601. Colombo, G., Remarks and additions to a preceding note (in Italian), *R. C. Semin. mat. Univ. Padova* **20**, part 1, 219–223, 1951.

Author generalizes some of his results published before [AMR **4**, Rev. 4069]. He shows, e.g., that the merostatic motion of a gyroscope is stable with respect to a noncyclic coordinate if a certain function V has an effective maximum. V is the difference of the potential of the disturbing force and a quadratic function of the constants of the integrals of momentum. In case that there is only one noncyclic coordinate, the merostatic motion is unstable if V has a stationary value and fulfills a certain additional condition. Gerhard W. Braun, USA

602. Heinrich, G., Experimental and theoretical investigation of the motion of a symmetrical gyroscope running upon a pin (in German), *Öst. Ing.-Arch.* **5**, 4, 322–339, 1951.

In order to examine the abrasion of the steel point of a symmetrical gyroscope running freely upon a spherical brass, several phases of the center-pin abrasion have been fixed microphotographically. Further, the speed of the starting gyroscope has been measured by photoelectric methods as a function of the time. The brake moment of the point friction, calculated in an earlier work, provides, connected with a mathematical arrangement of air friction, a good agreement with the measurement for all numbers of revolutions over 600/min. The situation corresponds to the state of dry friction: this is to be considered as a result of the high specific pressure. Also, the rising curves of the gyroscope axis, measured by observation in a gyrosectant, agree with theoretically deduced results. Discrepancies appearing in the case of low numbers of revolutions (under 600) are explained by strong dependence of the sliding friction coefficient upon the velocity in the reach of lower sliding velocities.

From author's summary by Herbert Bilharz, Germany

603. Malkin, I. G., On the theory of stability of regulated systems (in Russian), *Prikl. Mat. Mekh.* **15**, 1, 59–66, Jan.-Feb. 1951.

Using the second method of Lyapunov, author discusses the stability of the system $dy/dt = Ay + f(u)h$, where A is a constant matrix, h is a constant vector, and f is a scalar function of a linear function of the components of y. R. Bellman, USA

604. Oldenbourg, R. C., and Sartorius, H., Dynamics of automatic regulations [Dynamik selbsttätiger Regelungen] vol. 1, 2nd ed., (in German), München, Verlag R. Oldenbourg, 1951. 258 pp., 112 figs., 1 table. DM 26.

First edition of this book (1944) was reproduced in this country in 1948 by ASME. The present (2nd) edition reproduces the first with only minor corrections. It contains the general theory of linear regulators, with systematic use of the Laplace transform, and careful studies of a number of types of mechanical regulators. The technique used for continuous characteristics is extended to specific cases of piecewise continuous characteristics and to discontinuous, step-by-step regulators. Authors consider that only nonoscillatory behavior can be satisfactory (p. 63) and bases on this preliminary condition the definition of his quality factor. Authors' style is very clear; many of the illustrations represent relationships between nondimensional variables, and will be appreciated by the specialist.   P. Le Corbellier, USA

605. Blaquière, A., Extension of the Nyquist theory to the case of nonlinear characteristics (in French), C. R. Acad. Sci. Paris 233, 5, 345–347, July 1951.

Consider a vacuum-tube oscillator in which the amplitude of oscillation a is finally stabilized by a slight nonlinearity in a tube characteristic. Assume that a certain dynamic variable V is governed by a differential equation of the form $(H_1)V + (H_2)V^2 + (H_3)V^3 = 0$, where (H_1), (H_2), and (H_3) are differential operators. If there exists a solution of the form $V = a \exp(j\omega t)$ and if, as the author assumes, (1) all terms involving derivatives of a are negligible, and (2) only terms involving the fundamental frequency need be considered, then the defining equations become $(H_1) + (B3a^2/4)(H_3)|V = 0$. Now define functions H_1, H_3, and H, where H_1 is (H_1) with each differential operator d/dt replaced by $j\omega$, and $H = H_1 + H_3$. Author proposes that the stable final amplitude is given by that value of a which makes a Nyquist plot of H pass through the origin. Author points out that if Nyquist plots of H_1 and H_3 are made, then a Nyquist plot of H is easily generated for any value of a for any point on H_1 and must move along a line parallel to the radius vector of the corresponding point in H_3 as a grows in value. When power is first applied to the tube, $H_1 = 0$ and ω will be complex. The real part of this root (say ω_1) gives the initial rate of oscillation. In order that the initial rate shall be the same or the final, it is necessary that the phase angle of H_1, evaluated at ω_1, be exactly opposite to that of H_3.   Horace M. Trent, USA

Vibrations, Balancing

(See also Revs. 670, 714, 856, 909)

606. Heilig, R., Torsional and bending vibrations of thin walled beams with arbitrary open cross section under initial loads (in German), Ing.-Arch. 19, 4/5, 231–254, 1951.

This is a basic paper on theory of elasticity, developing fundamental relations of vibrating, open, thin-walled beams having arbitrary cross section and initial loads. Author distinguishes between "geometrical coupling" produced by noncoincidence of shear and gravity centers and measured as "shear center distance," and "mechanical coupling" from eccentricities of external loads. The hinged-hinged beam is considered as one example. Principal results: The shear center distance influences a triple infinity of characteristic frequencies if the beam cross section has no axis of symmetry, only a double infinity if the cross section has one axis of symmetry. (Note: The triple infinity in rectangular beam would refer to two bending vibrations, one in torsion.) Great wealth of detail and unconventional nomenclature make this hard reading.   R. M. Rosenberg, USA

607. Rost, J., Experimental determination of generalized masses of a system with mixed friction (in French), Rech. aéro. no. 22, 39–44, July-Aug. 1951.

Simple practical means are described for separating the effects of viscous and solid (Coulomb) friction during analysis of vibration-test data on lightly damped structures.

Assuming that structure is excited in a pure normal mode and that solid frictional forces are small enough to be replaced with their first Fourier-series components, author shows their only effect on resonant forced vibration to be an amplitude reduction. Their influence on damped free motion is well known to be a constant back-and-forward shift of the origin of exponential decay after each half cycle of damped sinusoid, and this behavior becomes basis of graphical scheme for estimating solid friction. Accuracy is improved by least-squares procedure.

Application to fuselage-torsion mode of ground vibration of an airplane demonstrates presence of both solid and viscous friction and separates them with satisfactory precision.

                    Holt Ashley, USA

608. Kneser, H. O., On the damping of vibrating cylindrical rods by the ambient medium (in German), Z. angew. Phys. 3, 3/4, 113–117, 1951.

The influence of the ambient medium (air) on the attenuation of the vibrations of a cylindrical rod is determined experimentally, including high modes of vibrations in bending, torsion, and tension.

Author finds that one part of the damping, apparently due to friction, is proportional to the square root of the pressure, and that another part due to radiation is proportional to the pressure, all in agreement with theory. For torsional vibrations, the first part is prominent and for bending and tension the second.

Paper contains tables where logarithmic decrement is given for the different types of vibrations.

                    Frithiof I. Niordson, Sweden

609. Voelz, K., Damping of vibrating bodies through friction with the ambient medium (in German), Z. angew. Phys. 3, 5, 185–187, 1951.

Approximate expressions for damping are derived for the case of a cylinder vibrating in an ambient medium. Following assumptions are made: (1) Ambient medium is incompressible; and (2) boundary layers of the medium vibrate with the body at its velocity.

With these assumptions, and a consideration of the loss of energy of the vibrating body and the work of friction at the surface of the body, author calculates the logarithmic decrement for longitudinal, flexural, and torsional vibrations of the cylinder. The results are compared with those obtained by Kirchhoff for a sphere and derived from the differential equations of motion.

                    Wilhelm Ornstein, USA

610. Deffet, L., Considerations of vibration measurements made in the Quenast quarry (in French), Explosifs 4, 3, 123–130, July-Sept. 1951.

Amplitude, distance, and frequency data are given for blasts at the Quenast quarry. Various damage criteria, attempting to correlate damage to neighboring structures with displacement, acceleration, etc., are mentioned; energy density is favored.

                    Robert D. Specht, USA

611. Minorsky, N., On the nonlinear Mathieu oscillator (in French), C. R. Acad. Sci. Paris 232, 24, 2179–2180, June 1951.

Author discusses the existence and stability of approximately periodic solutions of the differential equation $\ddot{x} + [1 + \epsilon(A -$

$Cx^2) \cos 2t | x = 0$, in which A and C are constants, and ϵ is a small parameter. It is shown that no such solutions can exist if $A/C < 0$. If $A/C > 0$ there exist, in first approximation with respect to ϵ, stable periodic solutions. The "amplitude" $(x^2 + \dot{x}^2)^{1/2}$ and the "phase" $\arctan(\dot{x}/x)$ of these solutions are approximately equal to A/C and to $k\pi/2$, respectively, where $k = 0, 1, 2,$ or 3, depending on the initial conditions. The method used is the same as in two previous notes by the author [title source **231**, 1117–1119, 1950; **232**, 1060–1062, 1951].

Courtesy of Mathematical Reviews   W. Wasow, USA

612. Jacobsen, L. S., and Ayre, R. S., Hydrodynamic experiments with rigid cylindrical tanks subject to transient motions, *Bull. seism. Soc. Amer.* **41**, 4, 313–346, Oct. 1951.

Fluid-containing tanks were subjected to horizontal shaking-table motions of impulsive or of oscillatory nature, idealizing earthquake motions. Measurements were made of gravity waves produced, and equivalent mass and overturning moment of fluid. Parameters were type, intensity, and time scale of applied motion; tank size, depth of fluid, and clearance between fluid and tank cover. Summarized results are presented in dimensionless form with precautions for extrapolation to full-scale water and oil tanks. Effective hydrodynamic mass and height of its centroid are given as functions of ratio of fluid height to tank radius. When cover clearance to radius ratio is less than about 5%, effective mass is practically open tank value even for large amplitude waves. Characteristics of wave motion require more care in extrapolation, but are not important in sufficiently large tanks. Most of motion is due to first or "sloshing" mode.

Vincent Salmon, USA

613. Shatashvili, S. Kh., On steady vibrations for given external forces on the surface of an elastic body (in Russian), *Prikl. Mat. Mekh.* **15**, 5, 615–617, Sept./Oct.

This problem has been treated formerly by Kuskov [AMR **4**, Rev. 109]. It is reduced here to a different system of Fredholm equations, solvable approximately for finite regions and any frequencies.   J. R. M. Radok, England

614. Traill-Nash, R. W., The anti-symmetric vibrations of aircraft, *Aero. Quart.* **3**, part 2, 145–160, Sept. 1951.

Assumption is made that the fuselage vibration is pure torsional (bending of fuselage is excluded) and that wing and tail have torsional and flexural vibrations. Author uses the "lumped mass" representation of the system and matrix notation in deriving the eigenvalue equation. Procedure is the same as that used in a previous paper dealing with symmetric vibrations and described in AMR **4**, Rev. 3478.   Wilhelm Ornstein, USA

615. Parmakian, J., and Jacobson, R. S., Measurement of hydraulic turbine vibration, Ann. Meeting ASME, Atlantic City, Nov. 1951. Paper no. 51–A-88, 5 pp., 12 figs.

Paper describes measurement and method of elimination of severe vibration and cracking of the runner blades of a 94.7-rpm, 40,000-hp hydraulic turbine installed at the Parker power plant on the Colorado river. Strain gages, accelerometers, and pressure cells were mounted on the runner blades in various locations. Pressure cells located at the leading edge of the runner blades indicated that the vibration was not a result of excitation from individual streams from the guide vanes. Pressure cells located at the trailing edge of the runner blades indicated periodic pressure fluctuations of high magnitude. Production of a von Kármán vortex street from the blunt trailing edges was inferred as a possibility of the cause of the vibration. After trailing edges of the blades were modified to reduce the bluntness, vibration was sub-

stantially decreased and a 6.5% increase in the maximum output of the turbine was noted. Three other runners were modified in the same manner; after an operating period of five months, no further cracking of the blades had occurred.

Richard H. Kemp, USA

616. Mettler, E., On the stability problem of forced vibrations of elastic bodies (in German), *ZAMM*, **31**, 8/9, 263–264, Aug./Sept. 1951.

Author considers the coupling effect between longitudinal and transversal vibrations of a two-hinged straight beam, taking account of finite deformation. Introducing the subsequently confirmed assumption that the extension of the neutral line is a function of time only, and considering a sine-shaped deflection, he derives a nonlinear ordinary differential equation of the second order for the time-dependent amplitude, the same equation as for undamped forced vibrations of an oscillator with a restoring force, including third-degree terms of the dependent variable. From the known stability properties of this equation, conclusions are drawn as to the existence of jump phenomena, etc.

Folke K. G. Odqvist, Sweden

617. Raman, Sir C. V., The vibration spectra of crystals and the theory of their specific heats, *Proc. Indian Acad. Sci. (A* **34**, 3, 141–151, Sept. 1951.

Nonmathematical survey of author's reasons for rejecting the classical Born- von Kármán theory of lattice vibrations. Elementary analysis of normal modes of finite crystal is unsound, because even very small damping (i.e., anharmonicity) will prevent establishment of standing waves in large crystal: It is known that a crystal of rock salt cannot transmit light of certain frequencies: the analysis of infinite crystals by Brillouin deals with running waves, which are not strictly normal modes; if one considers a magnified model of a diamond lattice one sees that a localized disturbance, in which atoms belonging to the two interpenetrating Bravais lattices move in opposite directions, spreads only slowly; existing calculations of vibration spectra lead to a diffuse continuum, and "one may feel sure" that calculations for crystals containing molecules with internal degrees of freedom would lead to similar results, whereas the observed Raman spectra of crystals show sharp lines.

Reviewer believes that all observed effects are adequately explained by classical theory (except for cases in which temperature at which high-frequency vibrations become quantum-mechanically degenerate lies above temperature at which anharmonic terms for low-frequency vibrations become large), while author's theory rests on a confusion between properties of finite and infinite lattices.   F. R. N. Nabarro, England

Wave Motion, Impact

(See also Revs. 592, 612, 748, 771, 772)

618. Officer, C. B., Jr., On the existence of Uller's waves, *Bull. seism. Soc. Amer.* **41**, 4, 307–311, Oct. 1951.

Uller assumed that seismic waves are described by functions of the form

$$\psi = [A_1 \cos(\nu_1 t - \Phi_1) + A_2 \sin(\nu_1 t - \Phi_1)] \exp(\nu_2 t - \Phi_2)$$

where A_1, A_2, Φ_1, Φ_2 are functions of position. In the initial stages $\nu_2 > 0$, then $\nu_2 = 0$, and finally $\nu_2 < 0$. He considered that in an unexplained way this formula supersedes the physical equations of wave propagation. Author asks, more reasonably, whether it can be deduced from them, and asserts that it cannot. He also criticizes Uller's views on refraction and on group

velocity. Reviewer finds the arguments obscure, but agrees with the conclusion, rejecting Uller's point of view which appears to him founded on a misunderstanding of scientific method. Uller gives no physical reason for his formula, which therefore explains nothing if, by an explanation, is meant the correlation of observation with an adequate physical theory. For an earlier severe criticism of Uller, see J. Fuhrich [Gerlands *Beiträge z. Geophysik* 52, 229–230, 1938]; for a modern view of group velocity, see Jeffreys and Jeffreys ["Mathematical physics," 479–487, Cambridge, 1946].   Fritz Joseph Ursell, England

619. **Smith, A. G., McCaig, I. W., and Inverarity, W. M., Maximum impact pressures on seaplane hull bottoms,** *Aero. Res. Counc. Lond. curr. Pap.* 4, 37 pp., 1950.

Authors consider the problem of obtaining true peak pressures and mean maximum pressures over any area from the maximum pressure measured on a diaphragm. Mechanical pressure recorders were employed on three hulls of 3-ft beam and dead-rise angles of 10°, 20°, and 30°. Through the use of an "area" and "velocity" factor, reasonable agreement was obtained between theory and experiment for the high angle "stall-on" flight-path angle. The method merits attention but considerable work is indicated in obtaining similar agreement at low flight-path angles and the planing condition.   Ernest G. Stout, USA

620. **Gold, L., Ultrasonic wave propagation in materials:** A guide to theoretical results, Symp. Ultrasonic Test., *ASTM Spec. tech. Publ.* 101, 1–13, 1951.

Author gives nonmathematical summary of factors involved in ultrasonic pulse methods of investigating solids, and describes general technique, types of waves generated, sources of energy loss, etc. Good introductory bibliography is included.
                                        Robert W. Morse, USA

621. **Friedlander, F. G., On the half-plane diffraction problem,** *Quart. J. Mech. appl. Math.* 4, part 3, 344–357, Sept. 1951.

Author considers the diffraction of an arbitrary two-dimensional disturbance by a semi-infinite plane screen. The problem is reduced to that of finding the solution of two simpler problems (called I and II). It is shown rigorously that the solutions of these problems, if they exist, are given by certain formulas. The problem of existence is then outlined. The method employed is a modification of that used by Hadamard in his discussion of Cauchy's problem. A remarkably simple formula for M, the diffraction of the disturbance represented by Hadamard's "elementary" solution, is obtained. Finally, author compares his results with those obtained by Gunn [*Philos. Trans. roy. Soc. Lond.* (A) 240, 327, 1947], Evvard [*NACA TN* 1382, 1947], Ward [*Quart. J. Mech. appl. Math.* 2, 136, 1949], and Fox [*Philos. Trans. roy. Soc. Lond.* (A) 241, p. 71, 1948] by different methods. In particular, it is shown that the Laplace transform of the solution of the elementary problem agrees with the Green's functions derived from Sommerfeld's two-valued solutions of the wave equation.   I. N. Sneddon, England

Elasticity Theory

(See also Revs. 635, 637, 649)

622. **Parkus, H., On the extension of the Hamilton principle** on thermoelastic phenomena (in German), *Federhofer-Girkmann-Festschrift*, Franz Deuticke, Wien, 295–301, 1950.

Starting with the first law of thermodynamics, author assumes perfect elasticity of the body to assure reversibility of the process and applies the second law of thermodynamics. Defining the ex-

ternal work by kinetic and strain energy, author very skillfully expresses entropy and free energy as a function of temperature and strain tensor. The principle of minimum potential energy is then derived for a static load and a uniformly distributed temperature. With potential and kinetic energy as a function of time, displacement, and strain tensor, Hamilton's principle can be applied to a general case of an elastic body subjected to forces and heat, both varying with time.

By combining the equation of heat conduction and boundary condition, an additional variational principle is set up, which, together with Hamilton's principle, determines all temperature and deformation phenomena of an elastic body.
                                        Wilhelm Ornstein, USA

623. **Sternberg, E., and Sadowsky, M. A., On the axisymmetric problem of the theory of elasticity for an infinite region containing two spherical cavities,** Ann. Meeting ASME, Atlantic City, 1951. Paper no. 51—A-10, 9 pp.

Using the rotationally symmetric stress-function approach, authors treat the state of stress in an infinite elastic body with two spherical cavities of the same radius. At infinity, a uniform stress field is acting in the body, while the cavities are loaded by a system of tractions.

As the solution could not be given in a finite form, but as a series, the results require many numerical calculations, which are performed by authors for two special cases: (1) Surfaces of the cavities free from tractions; (2) the stress state at infinity being hydrostatic.   H. Neuber, Germany

624. **Arzhaníkh, I. S., The integral equations of the deformation vector of the statics of an isotropic elastic body** (in Russian), *Dokladí Akad. Nauk SSSR* (N.S.) 75, 783–786, 1950.

Let $u = (u_1, u_2, u_3)$ be the displacement vector of a three-dimensional isotropic elastic body occupying a bounded domain Q possessing a smooth boundary surface S. The displacement u satisfies the system of differential equations of equilibrium:

$$\mu \nabla^2 u + (\lambda + \mu) \nabla \, \text{div} \, u + f = 0$$

where λ and μ are Lamé's constants of elasticity and f is the body force. Assuming that the domain Q possesses a Green's function G for the Dirichlet problem, author obtains, in terms of G, an integral equation satisfied by the displacement u in the case of the first boundary-value problem of elasticity, when u is prescribed on the boundary S. Analogously, assuming that the domain Q possesses a Green's function N for Neumann's problem, an integral equation satisfied by u, the kernel of the integral equation involving N, is obtained when u is a solution of the second boundary-value problem of elasticity, when the surface forces are prescribed on S. The derivation of the integral equations is based on two integral equations for the dilatation, div u, obtained previously [see AMR 4, Rev. 3801].
Courtesy of Mathematical Reviews   J. B. Diaz, USA

Experimental Stress Analysis

(See also Rev. 640)

625. **Ballet, M., and Mallet, G., On the use of ethoxyline resin in three-dimensional photoelasticity for the freezing technique** (in French), *C. R. Acad. Sci. Paris* 233, 16, 846–847, Oct. 1951.

A recognized disadvantage of the freezing technique is the excessive amount by which a model must be strained to give a large enough optical effect for a useful stress analysis to be made. In a recently introduced material, the strain required for a given number of fringes is stated to be only one fifth of that for Bakelit-

BT 61 893 and one third of that for Fosterite. The new resin should therefore increase the value of this already useful method of three-dimensional stress analysis. The material can be cast and the time-edge effect is said to be negligible.

A. F. C. Brown, England

626. Hickson, V. M., Photoelastic determination of free boundary stresses on frozen stress models by an oblique incidence method, *Brit. J. appl. Phys.* 2, 9, 261–269, Sept. 1951.

Paper deals with the determination of surface stresses in three-dimensional photoelasticity. The particular problem investigated is the determination of the factor of stress concentration in a grooved circular cylinder in compression. Author follows the procedure of removing thin slices from models with "frozen stresses" and viewing them at oblique incidence. Instead of using tangential slices, he prefers to use normal slices. The reason given for this preference is that a closer approximation to the true surface stresses can be obtained from normal slices than from tangential slices. The paper is notable for the great pains taken to attain a high degree of accuracy. Slices as thin as 0.020 in. were used and thicknesses were measured to within 2μ by using a traveling microscope. Retardations were determined to within 0.005 fringes by combining compensation with photoelectric equipment. The retardations were observed to within 0.003 in. from the boundary.

Reviewer wishes to observe that retardations very close to the boundary can also be determined by removing several thousandths of an inch from the free surface of tangential slices and measuring differences in retardation at incidence normal to the free surface.

Results obtained by author show remarkably little scatter and a very high degree of consistency, reflecting the precision methods employed. Author demonstrates that the method of oblique incidence can yield results of high accuracy. The difference between factors of stress concentration for the grooved cylinder obtained by oblique incidence differ by no more than 6% from those obtained from normal incidence.

Author states that correction for "rotational" effects could not be made because his polarizer and analyzer could not be turned independently. At the risk of being dubbed "old-fashioned," reviewer wishes to point out that this is a consequence of the trend toward over-mechanization of polariscopes in the attempt to achieve a higher "efficiency."   M. M. Frocht, USA

627. Meier, J. H., Experiences in the application of electric strain gages (in German), *Schweiz. Bauztg.* 69, 37, 38; 516–519, 527–531; Sept. 1951.

Paper deals with methods developed by author for stress analysis and performance analysis of large machines with emphasis on application of SR-4 strain gages. Bridge circuit (a-c or d-c), amplifying equipment, and rapid-response recorders are discussed. (A bridge oscillator and amplifier achieving much the same result as that employed by author is described by Burr [*Electronics* 22, p. 101, 1949].) Method is given for computing strain response of 4-legged strain-gage bridge, knowing individual gage characteristics and elastic constants of carrier metal (restricted to condition that gage axes and principle strain axes coincide). Other devices described include solenoid pickups, an ingenious motor transducer for measuring acceleration and velocity, and an elaborate mobile laboratory for stress analysis in the field. The many practical suggestions should prove useful to the engineering experimenter.   J. T. Bergen, USA

628. Carter, J. W., Stress concentration in built-up structural members, *Amer. Rly. Engng. Assn. Bull.* 53, 495, 1–34, June–July 1951.

Stress concentration is one of the main criteria in predicting

structural behavior of members subjected to repetitive loads. This report describes tests carried out to determine stress concentrations in plates in vicinity of rivet or bolt holes under varying conditions of pitch, gage, edge distance, bearing, and clamping force. Strain measurements were carried out with SR-4 gages and by photoelastic methods. The nature of material used for photoelastic models is not stated.

Conclusions drawn from the tests are: (1) Stress concentrations at sides of open holes in plates will vary within relatively small limits if conventional spacing of 3-hole diam and edge distances of $1\frac{1}{2}$ diam is maintained. The value of this stress concentration will vary between three and four. (2) Stress concentrations at sides of holes with pins in bearing in double shear will be higher than for open holes. In centrally loaded plate with diam-of-hole over width-of-plate ratio of 1 to 3.8, the stress is five times average stress on gross section of the plate. Eccentric loading leads to higher values. (3) Plates connected by pins bearing in single shear have stresses at sides of holes of 20 to 40 or more times average stress on gross section of plate, when load is applied centrally with respect to width of plate, as compared to five for a similar plate subjected to bearing in double shear. With bending or load applied eccentrically with respect to width of plate, stress concentration probably would be higher. (4) In joints connected by high clamping bolts, stresses inside hole produced by clamping applied through washer only are compressive. Axial stresses on plate at outside edge of washer are tension stresses, equal to about one-fifth those produced at edge of hole on similar plate connected by bearing pins in single shear. (5) Members with rivets or pins which may come to bearing due to loss of clamping force, and especially those with rivets or pins in single shear such as floor-beam hangers, may be expected to have a reduced fatigue strength. (6) Connecting two or more structural members by high clamping bolts will reduce elastic stress concentration and improve fatigue strength of members connected, over the fatigue strength of plates connected by rivets or pins in single shear or plates with open holes. (7) The fatigue strength of members connected by high clamping bolts may be further increased by using washer which is tapered, thinner toward its outside edge, to distribute the compressive stresses between washer and plate more uniformly over washer area. (8) Clamping force greater than that needed to resist slip of member will increase stress at edge of nut or heavy washer, for a given design of washer, and, therefore, cause the connected member to have a decreased fatigue strength.   S. K. Ghaswala, India

Rods, Beams, Shafts, Springs, Cables, etc.

(See also Revs. 606, 671, 677, 679, 680, 800)

629. Pudovkin, M. A., On the computation of the axis of a bent beam (in Russian), *Dokladi Akad. Nauk SSSR* (*N.S.*) 77, 6, 993–995, Apr. 1951.

Author considers a beam with arbitrary supports at the ends and arbitrary loads in one plane. The loads, if discontinuous, have a finite number of discontinuities n. By integrating n times the general differential equation of the bending curve and transforming the multiple integrals, the solution of the equation in terms of Stieltjes integral is obtained. The solution can also be expressed as a Volterra integral equation. The shearing force, the bending moment, and the angle at the support can be evaluated from the solution by differentiation.

Courtesy of Mathematical Reviews   T. Leser, USA

630. Nowinski, J., Flexure of beams by terminal loads (in Polish), *Arch. Mech. stos.* 2, 89–105, 1950.

Author considers an isotropic homogeneous cantilever beam of

uniform cross section, fixed at one end, $z = 0$, and loaded at the other, $z = l$, by forces equivalent to a single force $P(P_x, P_y, 0)$, acting in the point (x_W, y_W, l). The z-axis coincides with the central line of the beam, which is the locus of the centroid of a cross section. The body forces are assumed to be zero. The displacements (u, v, w) are found from the general formula by substituting the constants defined from the above conditions. Author derives the expression for the angle of rotation of an element of a cross section in the plane of this cross section, which he calls the local rotation. The local rotation of an element containing the centroid is called the mean rotation. The local rotation depends on the fixing conditions and on the position of the load point. If the load point coincides with the position of the center of flexure, the mean amount of rotation for every cross section is constant and generally different from zero. The mean amount of rotation may equal zero only if the fixed points are suitably chosen or if the beam is subjected to pure torsional couple. The axis of local pure translation which is the locus of a point which is only translated, and the axis of pure rotation which is the locus of a point which is not translated in the plane of a cross section, are found. These loci, in general, are some space curves; when the beam is subjected to pure torsional couple, the axis of pure rotation is the straight axis of twist.

Courtesy of Mathematical Reviews   T. Leser, USA

631. Coates, R. C., Solving beam problems by relaxation methods, *Engineering* **172**, 4472, 456–457, Oct. 1951.

The differential equation of the deflection curve of a beam is replaced by appropriate difference equations. Solution of resulting simultaneous linear equations by relaxation method gives shape of elastic line. Examples treated are the deflection lines of simply supported prismatic and nonprismatic beams, and the end moment influence line (by Mueller-Breslau's theorem) of a symmetrical nonprismatic beam fixed at both ends.

G. Sved, Australia

632. Gorgidze, A. Ya., The torsion and bending of composite bars with slightly curved axes (in Russian), *Trudy Tbiliss. Mat. Inst. Razmadze* **17**, 95–130, 1949.

Paper contains an approximate solution of Saint Venant's torsion and flexure problems for a slightly curved compound beam made up of materials with different Young's moduli but with like Poisson's ratios. The cross section S of the beam consists of an exterior simple closed contour L_0, containing within it a set of nonoverlapping regions $S_i(i = 1, 2, \ldots, m)$ bounded by simple closed contours L_i. The contours $L_i(i = 0, 1, \ldots, m)$ are given by equations of the form $f_i(x + kz^2, y) = 0$, where the coordinate z is directed along the length of the beam, so that the axis of the beam, in the undeformed state, is a parabola whose curvature is determined by the small parameter k. The lateral surface of the beam is free of stress and the external forces act in the end $z = l$ of the beam.

The components of displacement are assumed to be continuous throughout the cross section S. If the coordinates ξ, η, ζ defined by $\xi = x + kz^2, \eta = y, \zeta = z$ are introduced, the equations of contours L_i assume the forms $f_i(\xi, \eta) = 0$. A change of variables in the elastostatic equations leads to a set of linear equations in the space of the variables ξ, η, ζ provided that one neglects terms of the order k^2. The paper contains solutions of such equations for the problems of extension, torsion, pure bending, and flexure by a transverse force, with illustrations of the first three of these problems. Illustrations deal with a slightly bent circular beam reinforced by a concentric circular core. The treatment follows N. I. Muskelishvili [*Izv. Akad. Nauk SSSR, Fiz. Nauk* (7), 567–945, 1932] and P. Riz [*Dokladi Akad. Nauk SSSR* **24**, 110–113, 229–232, 1939].

I. S. Sokolnikoff, USA

633. Popov, E. P., Successive approximations for beams on an elastic foundation, *Amer. Soc. civ. Engrs. Proc.* Separate no. 18, May 1950 = *Trans. Amer. Soc. civ. Engrs.* **116**, 1083–1095, 1951.

Paper demonstrates application of Stodola-Vianello iterative procedure with graphical integration to determine the elastic curve of a finite beam resting upon an ideal elastic foundation. Effects of eccentricity of loading, variable rigidity of beam, and nonuniformity of foundation modulus are shown to be easily included.

First cycle assumes soil pressures as under an infinitely stiff beam. To compensate for tendency to overshoot, author uses average of initial and final values of each cycle as basis for succeeding cycle. Reviewer believes that averaging technique may prove undesirable for certain ranges of beam rigidity-to-foundation modulus ratio and that experienced engineers might use a more realistic first assumption. Author's rules, however, standardize the procedure and thus may be justified.

John E. Goldberg, USA

634. Schade, T., Calculation of the pressure distribution of a loaded beam on elastic foundation (in German), *ZAMM* **31**, 8/9, 272–274, Aug./Sept. 1951.

Equating local deflection of beam and foundation, author gives a linear integral equation for the reaction of the foundation, which can be transformed, using Hamel's solution, to a system of linear equations. The results of three examples are given in graphs.

W. L. Esmeijer, Holland

635. Sheng, P. L., Note on the torsional rigidity of semicircular bars, *Quart. appl. Math.* **9**, 3, 309–310, Oct. 1951.

Author finds the exact value for the torque T transmitted by a semicircular shaft of radius a, shearing modulus μ, and angle of twist α, to be

$$T = \mu\alpha a^4(\pi^2 - 8)/2\pi = 0.297556\mu\alpha a^4$$

which is to be compared with published values $0.296\mu\alpha a^4$ or $0.2966\mu\alpha a^4$. The closed form of the result is found by obtaining exact sums of several series.   D. L. Holl, USA

636. Abramyan, B. L., and Dzhrbashyan, M. M., On torsion of shafts with variable cross section (in Russian), *Prikl. Mat. Mekh.* **15**, 4, 451–472, July–Aug. 1951.

Boundary consisting of two circular cylinders having the same axis but different radii is considered. Twisting couples are applied by means of tangential tractions exerted upon portions of cylinders. According to theory due to J. H. Michell (and A. Föppl) the displacement at any point is directed at right angles to the axial plane passing through the point so that stress components can be expressed in terms of a single function. Dividing the shaft into three portions (one of them belonging to both cylindrical bodies) three stress functions are found in the form of series of Bessel functions, coefficients of these series being roots of an infinite system of linear equations. Numerical example is given illustrating application of theory. Results are not compared with earlier analytical and experimental research, e.g., with the approximate solution obtained by F. A. Willers.

J. M. Klitchieff, Yugoslavia

637. Mitra, D. N., Torsion and flexure of a beam whose cross section is a sector of a curve, *Bull. Calcutta math. Soc.* **43**, 1, 41–45, Mar. 1951.

Paper extends work on the circular sector, reviewed earlier [AMR **4**, Rev. 2403], to the more general case. Method followed is the same as in previous paper, but formulas now contain the unknown mapping function corresponding to the arbitrary bounding curve.   W. S. Hemp, England

638. Anderson, C. G., Flexural stresses in curved beams of I- and box-section, *Instn. Mech. Engrs. appl. Mech.* **163,** (*W.E.P.* 62) 295–306, 1950.

Failures experienced in practice caused this investigation which indicates that curved beams of I- and box sections, owing to large secondary bending stresses (in transverse direction), do not develop their full strength. Theoretical and experimental investigation is given; the first being divided into two parts: a theory of flexure of full sections, refining the well-known work of Winkler, Grashof, Martin, Gough and others, and a theory of distortion of thin-walled sections based on the investigation of von Kármán, Timoshenko, Bach, Steinhardt, and others. The Winkler-theory (plane strain and a hyperbolic distribution of circumferential stresses) is in satisfactory agreement with a plane-stress analysis given by author. From the designer's point of view, the flange distortion is more important: Author extends the classical theory to a box beam with an additional web. Tables, figures and graphs, a special design chart, design comments, and a concluding survey summarize the results for design purposes.

K. Marguerre, Germany

639. Benischek, J., General calculation of stresses in a circularly curved tube nonuniformly heated from outside and loaded by an absolute internal pressure greater than atmospheric (in German), *Öst. Ing.-Arch.* **5,** 2, 117–129, 1951.

Tube considered is in the form of a torus, and is heated so that the temperature of the outside of the tube varies from the inside to the outside of the torus according to a cosine law, while the temperature of the inside of the tube is constant. The temperature variation through the wall of the tube is assumed to be linear. The usual assumptions in theory of thin shells are made, and the problem is reduced to that of solving a single linear, nonhomogeneous, differential equation. Solution is obtained in form of a power series, permitting evaluation of stresses. Results are believed applicable to curved portions of tubes in steam boilers. Numerical calculations in an example using typical boiler conditions are said to show that radial, circumferential, shearing, and bending stresses, all are important; the reader, however, has no opportunity to judge for himself, for the specific values thus obtained are, unhappily, not given. A. D. Topping, USA

640. Aubaud, J., A photoelastic study of stresses in the fillets of a pair of spur gears (in French), *Rech. aéro.* no. 22, 33–38, July-Aug. 1951.

Paper describes photoelastic determination of fillet stresses in a particular gear pair, the Hispano 95 spur reduction. Double-size Plexiglass models were used, statically loaded. Various fillet shapes were investigated on the gear wheel only, all other variables remaining unchanged. Semicircular fillets reduced stresses 20% compared with generated trochoidal fillets. A total of 30% improvement was shown with parabolic shape. Limited experiments on the pinion showed a 20% stress reduction with parabolic fillets compared with the trochoidal.

Ewen M'Ewen, England

641. Popov, E. P., Nonlinear problems of the statics of thin rods [Nelineinie zadachi statiki tonkikh sterzhnei], Moscow-Leningrad, Gosud. Izdat. Tekh.-Teor. Lit., 1948, 170 pp., 1 plate.

This is a self-contained monograph devoted to a systematic exposition of solutions of a broad class of elastostatic problems on large deflections of thin rods and flat blades. The theory, including a careful derivation of the Kirchhoff equilibrium equations, is contained in the first chapter (49 pp.). The remaining two chapters (103 pp.) deal with the applications of the theory to diverse problems on deflection and stability of thin rods deformed in one

plane. The book summarizes all significant published results on problems of this category up to 1948. A distinguishing feature of the work is that the exact solutions are presented in a form suitable for numerical computation, so that they can be used directly by a stress analyst. The bibliography contains 85 items, the earliest of which is dated 1867. I. S. Sokolnikoff, USA

642. Sourochnikoff, B., Strength of I-beams in combined bending and torsion, *Amer. Soc. civ. Engrs. Proc.* Separate no. 33, Sept. 1950 = *Trans. Amer. Soc. civ. Engrs.* **116,** 1319–1336, 1951.

S. Timoshenko ["Theory of elastic stability," McGraw-Hill, 1936] discussed this problem by using the energy method and taking the angle of twist in the form of a cosine series, the first term of the series being sufficiently accurate for most engineering applications. Author makes some extensions and uses the deflection method, assuming the angle of twist to be a parabola. Equations for the torque for the cases of concentrated load at center and uniformly distributed load are approximated by parabolas and straight lines. Results are probably accurate enough for engineering applications, but are not presented in a form suitable for use in design. B. E. Gatewood, USA

Plates, Disks, Shells, Membranes

(See also Rev. 583)

643. Wei, C., The state of stress in toroidal and similar shells with azimuthal rings under torsionally symmetrical stress (in German), *Engng. Rep. nat. Tsing Hua Univ.* (A) **5,** 289–349, 5 plates, 1949.

The present paper deals with axisymmetrical deformations of thin elastic toroidal shells. The object is similar to the work of R. A. Clark [AMR 4, Rev. 4249]. Author obtains the asymptotic solution of the homogeneous differential equation in terms of Bessel functions of order $1/3$, not knowing of the tables of the modified Hankel functions of order $1/3$ and their derivatives [*Ann. Computation Lab., Harvard Univ.* **2,** 1945]. The asymptotic particular solution of the nonhomogeneous equation which is the crucial part of the work appears in a form which appears to be more complicated than the corresponding result in Clark's work. As example of application, the author calculates the stresses in a complete toroidal shell which carries equal and opposite line loads in the direction of the axis of the torus.

E. Reissner, USA

644. Lur'e, A. I., On the equations of the general theory of elastic shells (in Russian), *Prikl. Mat. Mekh.* **14,** 5, 558–560, Sept.-Oct. 1950.

In the classical theory of elastic shells, the three-dimensional problem is reduced to a two-dimensional one through a specification of eight forces and moments which are statically equivalent to the distribution of the normal stresses σ_1 and σ_2 and the shear stresses $\tau_{12} = \tau_{21}$ along the lines of curvature in the middle section. Four of these quantities, S_1, S_2, H_1, and H_2 are connected by the equation: $S_1 - S_2 + H_1/R_1 - H_2/R_2 = 0$ and are specially introduced to account for the shear stress τ_{12}. This note shows that in all equations of the elastic theory of shells these four quantities can be replaced by two quantities S and H given by:

$$2S = S_1 + S_2 - H_1/R_1 - H_2/R_2, \quad 2H = H_1 + H_2$$

Further, within the limits of accuracy of the thin shell theory, S and H are given by:

$$S = \frac{Eh}{2(1+\nu)}\gamma, \quad H = -\frac{Eh^3}{12(1+\nu)}\omega^*$$

where γ and ω^* are, respectively, the shear strain and the rotation of the middle surface.

Courtesy of Mathematical Reviews   H. I. Ansoff, USA

645. Münz, H., An integration method for the calculation of bending stresses of axisymmetrical shells under axisymmetrical load. II. Communication (in German), *Ing.-Arch.* **19**, 4–5, 255–270, 1951.

The homogeneous part of the differential equations found in part I [AMR **4**, Rev. 4107] is reduced, by substitution of complex variables and neglect of Poisson's ratio and the relative change of the thickness of the shell, to two systems of two linear differential equations of the first order. With these simplifications, allowable when the meridian tangent does not fall steeply with the axis of the shell, the evaluated equations are easy to integrate numerically.

With an iteration, the influence of the neglected terms can be accounted for. Particular integrals are found with the method of the variation of constants. After an application of the theory on two examples (tubes with a particular form of the meridian section), an integration method is given for the flat plate.

M. Botman, Holland

646. Clark, R. A., Gilroy, T. I., and Reissner, E., Stresses and deformations of toroidal shells of elliptical cross section. With applications to the problems of bending of curved tubes and of the Bourdon gage, Ann. Meeting ASME, Atlantic City, 1951. Paper no. 51—A-11, 12 pp.

Authors show that the membrane solution of H. Lorenz for a closed toroidal elliptic shell under uniform internal pressure forms the basis for a theory which considers bending—the results depending on two parameters b/c and bc/ah, where b, c are semi-axes of section, a the radius of the tore measured to the center of the section, and h the wall thickness. A trigonometric series solution is developed for bc/ah small while asymptotic integration is used for bc/ah large—the two solutions overlapping to cover the entire range of bc/ah. Authors show that as bc/ah vanishes, the maximum stress is bending since the direct stress vanishes. For bc/ah increasingly large, the reverse situation holds. The membrane solution is shown to be correct only if bc/ah is sufficiently large.

The above results are applied, with some modification, to a curved tube. Expressions for the effective stiffness (EI) are developed. The stiffness depends on both parameters. Curves show the values of both parameters where the asymptotic and series solutions agree. The results are also applied to a Bourdon tube where a moment correction is necessary. Curves show, in particular, both series and asymptotic solutions for: (1) Ratio of bending moment based on direct stress to corresponding moment from membrane theory; (2) ratio of maximum bending stress to corresponding stress in cylindrical tube. A formula proposed by Lorenz for the change in curvature is shown to be illogical. For large bc/ah, simple approximate expressions are derived.

Additional curves and discussion compare the results with the membrane solution for the thin elliptic shell and the bending solution for the straight elliptic cylinder. The establishment of definite limits of validity for the membrane solution will be welcomed by engineers. Authors suggest the analysis be extended to smaller values of the parameter b/c and to cross sections other than elliptical.

Herman A. Lang, USA

647. Jung, H., Pressure distribution under elastically supported ring plates (in German), *Z.A.M.M.* **31**, 8/9, 279–280, Aug./Sept. 1951.

Article complements an earlier paper where the mathematical basis is treated [AMR **4**, Rev. 1038]. A finite plate is assumed to rest without friction on a semi-infinite elastic foundation and subjected to rotationally symmetric load.

Stresses and displacements are given, using Love's displacement function for solving the given bipotential equation. Displacement function is obtained by multiplying each integral of the bipotential equation, all of the same parameter, by another suitable function of the same parameter, adapted to the actual deformation pattern using Hankel transforms. Terms are adequately summed up and integrated with respect to the parameter. A linear equation system is obtained from the boundary and equilibrium conditions, enabling the unknown constants of the assumed function to be obtained. No numerical results are given.

Sven T. A. Ödman, Sweden

648. Levi, F., Influence surfaces and adaptation phenomena in plane plates (in Italian), *G. Gen. civ.* **88**, 5, 326–335, May 1950 = *Ric. sci.* **20**, 4, 482–486, Apr. 1950.

See AMR **5**, Rev. 71.

649. Dzhanelidze, G. Y., On the theory of thin and thin-walled rods, *NACA TM* 1309, 18 pp., Oct. 1951.

See AMR **4**, Rev. 3919.

650. Griffith, G. E., Stresses in a two-bay noncircular cylinder under transverse loads, *NACA TN* 2512, 34 pp., Oct. 1951.

Paper gives an analysis, involving simultaneous solution of eighth-order differential equations, for stresses in a two-bay noncircular cylinder, enclosed between ring bulkheads rigid in their planes with the middle flexible ring subjected to concentrated and distributed loads. The noncircular fuselage is approximated by using circular sections of different radii and joining the sections at points of tangency. Shear and axial deformations of the loaded ring are neglected, but eccentricity of ring and sheet is included. Two numerical examples given agree with previously published circular shell data.

Clarence B. Matthews, USA

651. Lepik, Yu. R., Two remarks on the theory of stability of plates beyond the elastic limit, taking account of the compressibility of the material (in Russian), *Prikl. Mat. Mekh.* **14**, 5, 553–557, Sept.-Oct. 1950.

A. A. Ilyushin has discussed the problem for incompressible materials in terms of the theory of small elastic-plastic deformations. Present paper discusses the validity of Ilyushin's results for compressible materials. It is shown that the solution under the assumption of incompressibility approaches the true solution as the difference between the yield stress and the buckling stress increases. On the other hand, the solution based on the assumption of small plastic deformation can be used only so long as the buckling stress does not exceed the yield stress by more than 3%. The second part of the paper formulates four conditions under which the relative thickness of the plastic zone remains constant in compressible materials. It is shown that the assumption of incompressibility leads to an overestimate of the relative thickness.

Courtesy of Mathematical Reviews   H. I. Ansoff, USA

652. Popov, S. M., On the cylindrical form of the loss of stability of plates beyond the yield point (in Russian), *Prikl. Mat. Mekh.* **14**, 5, 543–552, Sept.-Oct. 1950.

The problem was considered before by A. Ilyushin ["Plasticity," OGIS, 1948] and leads to one transcendental equation and two quadratures. A practical determination of the stiffness or the critical load, the elastoplastic boundary, and the unloading zone is rather difficult, since the upper limit of integration de-

pends on the transcendental equation, which in turn is connected to one of the integrals. Ilyushin gives upper and lower bounds for the ratio of the thickness of the plastic range to the thickness of the plate, and concludes that the buckling load would not differ much in this approximate treatment. Present paper shows that the bounds mentioned do not permit such a conclusion. The critical load, indeed, does not differ by more than 6%. The boundaries of pure plastic deformation are determined and the relative thickness of the plastic range is found.

George Herrmann, USA

Buckling Problems

(See also Rev. 685)

653. Massonnet, Ch., Experimental investigation of the resistance to buckling of the web of solid web girders (in French). *Bull. Centre Étud. Constr. Génie civ. Hyd. Fluviale* **5**, 67–240, 1951.

Results are given of a test program in Belgium, to establish rules for designing solid web girders on the basis of safety and economy. A large portion of the paper is devoted to a detailed review of the theoretical literature on plate buckling. Sections are also devoted to results of recent tests by other investigators and to a review of existing specifications in several countries. The tests themselves are described in great detail. Southwell's method is found to be the most satisfactory for the nondestructive tests. The web girders are also tested to failure. Author concludes that the theoretical critical load, as calculated by Timoshenko's formulas for a simply supported plate, should be multiplied by a factor of safety of 1.35 for buckling due to shear and by 1.15 for buckling due to bending. These coefficients give a suitably large factor of safety with respect to the ultimate load.

Harold Lurie, USA

654. Woinowsky-Krieger, S., On the buckling safety of square plates with laterally displaceable boundaries (in German), *Ing.-Arch.* **19**, 3, 200–207, 1951.

Author calculates the buckling load of a rectangular plate, laterally compressed in its middle plane by forces uniformly distributed along two parallel sides which are free to move in a direction perpendicular to the plate, the two remaining edges being simply supported. Numerical results are presented in a figure for two values (0 and 0.3) of Poisson's ratio. Extension is made to plates of which one of the sides, along which the load is applied, is simply supported or clamped.

M. Kuipers, Holland

655. Marguerre, K., Stability of the cylindrical shell of variable curvature, *NACA TM* 1302, 64 pp., July 1951.

Translation from *ZWB Forschungsbericht* no. 1671, Sept. 1942.

656. Nowacki, W., Particular cases of buckling of plates (in Polish), *Arch. Mech. stos.* **2**, 107–122, 1950.

Author considers two cases: (1) A plate of infinite length freely supported on the edges, compressed at the edges by a distributed load q in the plane of the plate, and loaded by a distributed load p perpendicular to the plane of the plate. The plate is compressed additionally by two concentrated forces P and also has points of support between the edges. (2) A rectangular plate freely supported on the perimeter, compressed at two opposite edges by a distributed load q in the plane of the plate, and loaded by a distributed load p perpendicular to the plane of the plate. The case where an additional point of support is added is also considered. The solution of the partial differential equations in the form of infinite series is found from the given conditions,

and the critical values of P and q in both cases are determined. *Courtesy of Mathematical Reviews*   T. Leser, USA

657. Campus, F., Thoughts on M. Dutheil's method for the analysis of members in compression and bending (in French), *Ossature métallique* **16**, 1, 33–43, Jan. 1951.

Paper contains a systematic discussion of strut formulas, using the idea of initial eccentricity, as well as some consideration of the general problem.   W. S. Hemp, England

658. Dutheil, J., Discussion on the buckling of members compressed axially (in French), *Ossature métallique* **16**, 6, 315–325, June 1951.

In his two previous papers [*Inst. tech. Bât. Trav. publics*, Circ. I, p. 35, Sept. 1947; *Soc. roy. Belge Ingrs. Industr. Bull.* no. 3, May 1950] author recommended the method of assumed inaccuracies as a basis of column design. Advantages of this method—a more reliable basis for a proper factor of safety and better consistency with usual methods and principles of design in strength of materials—are well known, and it is due to the author that this method was adopted in new French specifications for the design of steel structures (Règles CM 1946). In the present article, in course of a discussion with F. Campus [see preceding review] and Ch. Massonnet [AMR **4**, Rev. 1094], author backs his proposal to compensate assumed inaccuracies by initial deflection of the column depending on the length of the column, the height of the cross section, and the maximum stress from bending at the outermost fiber, and carries out his way of calculating beams under combined action of bending and compression.

Dragoš Radenković, Yugoslavia

659. Salvadori, M. G., Numerical computation of buckling loads by finite differences, *Proc. Amer. Soc. civ. Engrs.*, Dec. 1949 = *Trans. Amer. Soc. civ. Engrs.* **116**, 590–624, 1951.

See AMR **3**, Rev. 1891.

660. Paris, A., Buckling of beams with solid or open webs (in French), *Bull. tech. Suisse Rom.* **77**, 17, 225–231, Aug. 1951.

Author develops approximate method for calculating lateral stability under bending moments of deep narrow reinforced concrete beams with solid or Vierendeel type webs. Standard methods are unsafe because concrete is unable to resist tension loads and torsional resistance cannot be of Saint-Venant type. For same reason, graphical or numerical methods have to be used. Paper is useful attempt to solve difficult problem but value would be enhanced by experimental check on necessary simplifying assumptions.   P. C. Dunne, England

661. Holt, Morris, and Clark, J. W., A study of end connections for struts, *Proc. Amer. Soc. civ. Engrs.*, Dec. 1949 = *Trans. Amer. Soc. civ. Engrs.* **116**, 223–245, 1951.

See AMR **4**, Rev. 1108.

Joints and Joining Methods

(See also Revs 661, 710)

662. Hartman, A., and Duyn, G. C., Comparative investigation of various types of riveted connections under a pulsating load. I (in Dutch), *Nat. LuchtLab. Amsterdam Rap.* M.1735, 7 pp., 4 tables, 10 figs., Jan. 1951.

Report deals with results of first part of comparative tests with various types of riveted joints used by aircraft industry. Final report (in English) will summarize all tests. Present tests are with two tyes of 17S rivets (round and countersunk rivet heads) of 3.1-mm diam in 0.8-mm 24S-T Alclad sheets; specimens had

15 or 16 rivets in two rows. Tests were made in Amsler high-frequency pulsator. Strain measurements were made to verify uniformity of stress distribution over width of specimen. No significant differences in static or fatigue strength were found for the two types of rivets. Test results are described in detail.

F. J. Plantema, Holland

663. **Kihara, H., Fatigue strength of spot-welded light alloy** joints, *Weld. Res. Suppl.* **16**, 10, 529s–536s, Oct. 1951.

Specimens were tested from plates of SDH (super Duralumin heat-treated) 0.032 to 0.08-in. thick with tensile strength of about 68,000 psi, statical shearing strength per welded spot being about 470 lb (0.032-in. thickness). First the effect of mean load upon the fatigue strength was proved. Shearing fatigue strength per spot, which has any mean load, can be estimated by testing both reversed fatigue strength and statical strength. The smaller the margin of plate in single-lapped joints, the lower is the shearing fatigue strength.

To ascertain the effects of pitch of spot welds upon the pulsating (zero-tension) fatigue strength, single-row and double-row lapped joints of 0.04-in. plates with 2, 3, 4, 5, and 6 spots and with pitches of 1.0, 0.67, 0.50, 0.40 and 0.33 in. were proved. Fatigue tests were also made on seam-welded joints to be regarded as zerolike pitch of spots. When the pitch p of the spots was larger than 0.5 in., the mean tensile fatigue stress σ of joint is shown by a hyperbolic curve

$$\sigma = n \cdot S_n/p \cdot t, \; (p > 0.5 \text{ in.}) \qquad [1]$$

where t is the thickness of plate, n the number of rows, and S_n the fatigue strength of one spot. When the pitch is smaller, the lapped joints will break all at the same load

$$\sigma = \sigma_n = \text{const}, \; (p < 0.5 \text{ in.}) \qquad [2]$$

The numerical values of S_1, S_2, σ_1, and σ_2 are given in a table. Saturated pitches for fatigue are about 0.5 in. for single- and double-row joints, while for the statical, saturating pitches of double-row joint are always larger than that of the single one.

E. Siebel, Germany

664. **Cooke, B., Pipe joints for hydraulic power transmission**, *Instn. mech. Engrs. Proc.* **164**, 3, 308–314, 1951.

Hydraulic machinery suffers from the disadvantage of leakage of the fluid. Faulty pipe joints are one of the main offenders in this respect. In this paper, author describes some tests, carried out on a variety of pipe joints, to find a satisfactory one. A joint and joint ring were designed from the results of the tests.

From author's summary

Structures

(See also Revs. 632, 653, 660, 723, 860, 885)

665. **Pirlet, J., Statics of rigid-frame structures [Statik der rahmenartigen Tragwerke]**, Berlin, Springer-Verlag, 1951, vii + 168 pp., 80 figs., 5 tables. DM 24.

By combining the carry-over and distribution factors of the Hardy Cross method of moment distribution into a "degree-of-restraint" factor, author reduces the labor of analyzing continuous beams or rigid frame structures. His method is valid for structures with no sidesway and for systems in which each member ik is symmetrical so that the change in slope at i, due to moment M acting at k, is equal to the change at k when M is applied at i.

Computations may be arranged in tabular form and methods for determining exact and approximate values are presented.

Several factors are precomputed and given in tables for intervals so small as to preclude interpolation in many cases.

Nomenclature is logical but complicated. It is developed and explained clearly so that an engineer using the method frequently should readily master and retain its details. They cannot be summarized in brief and lucid form.

Illustrative examples, exact and approximate, indicate how to combine factors and how to use influence lines for typical solutions. The procedure appears advantageous for multistory, multibay structures since each operation may be represented by a tabular quantity to be evaluated by one skilled in the use of computing machines though not necessarily in the analysis of multiple indeterminate structures.

Joseph S. Newell, USA

666. **Lamberg, R., Frame structures with dependent angles of rotation of the members** (in German), *Bauingenieur* **26**, 8, 240–244, Aug. 1951.

Author presents a method of solution of elastic rigid frames composed of members of constant moment of inertia. The elastic and equilibrium equations are arranged in symmetrical matrix form in terms of the unknown joint and sidesway angle changes. Solution of the matrix is suggested by means of the Gauss algorithm. Several examples including an unsymmetrical trapezoidal frame are discussed, but no numerical solutions are indicated. It is doubtful whether the method offers any advantages over the conventional moment-distribution techniques.

John Morley English, USA

667. **Ewell, W. W., Three-dimensional displacement diagrams for space frame structures**, *Proc. Amer. Soc. civ. Engrs.* Separate no. 20, May 1950 = *Trans. Amer. Soc. civ. Engrs.* **116**, 809–827, 1951.

Author applies to space pin-jointed structures the construction of Williot-Mohr displacement diagram. The possibility of ubicating a joint in such a diagram depends upon the knowledge of the variations of the distances between the joint considered and other three, already ubicated. The ubication of such a joint should be obtained as the point of intersection of the three planes normal to the directions of the lines joining the joint considered with the other three. To begin the drawing of the diagram, it is necessary to know the displacements of three joints, or to make hypothesis modifying the vinculations of the joints in such a way that the previous condition is satisfied.

Then it will be necessary to go back to the real conditions without modifying the deformation of the structure, that is, with a kinematic movement. The concepts applied are the same as for a plane Williot-Mohr diagram, translated to space. (The previous description would be that of a plane diagram if in it the words "three" and "plane" were respectively replaced by "two" and "straight line.")

The graphical representation of what has been stated, using an orthographic and an iconographic plane, as author does, originates a very complex construction, even for the first example developed, which is quite simple.

The results of the first example and a part of those of the second are checked by the classical analytical expressions of virtual work, indicating a maximum difference between both procedures of 1%. In this respect, reviewer agrees with author's opinion, that "The concurrence of values by the two methods is necessarily dependent upon the degree of accuracy with which the three-dimensional diagram is drawn," and adds that the degree of accuracy author has taken to obtain such a small error after so complex a graphical construction must have been very high, perhaps greater than allowed in a professional office. In this respect the scale employed, which is not indicated, must be of great importance.

Practical observations and suggestions formulated by W. Wat-

ters Pagon in the discussion, to lessen the confusion of the graphical work, seem to reviewer very useful.

Arturo J. Bignoli, Argentina

668. Baker, J. F., Behavior and conception of metallic frames (in French), *Rev. Soudure* **7**, 3, 128–146, 1951.

Author describes progress that has been made on developing a rational and usable method of structure analysis based on conditions for failure (*limit design*) as compared to the more common assumption of elastic behavior. He points out those fields which are most in need of further development. A brief summary is given of the not entirely successful attempts of the Steel Structures Research Committee (British) from 1929 to 1936 to devise an improved and not excessively complicated method of elastic analysis. From 1936 to 1939 and since World War II, much progress has been made in the field of plastic analysis. Items discussed are: Simple bending, with and without longitudinal force (but without instability), action of portal frames, shear combined with flexure, concentrated loads, strain hardening, lateral instability, repeated loads, deflections, and complicated structures, especially multistory buildings.   M. P. White, USA

669. Krall, G., On new construction methods in bridge building (in German), *Schweiz. Arch.* **17**, 9, 257–268, Sept. 1951.

Paper describes several arched bridges of reinforced concrete erected in the Melan system. Sequence of concreting is directed so as to make possible a common action of the Melan scaffolding plus first-poured parts, to support parts concreted thereafter. Creep is taken into account and some (known) formulas are quoted.   H. Craemer, Germany-Egypt

670. Delpuech, P., Dynamic flexure and oscillations of bridges (in French), *Ann. Ponts Chauss.* **121**, 1, 2, 3; 1–41, 225–247, 321–344; Jan.-Feb., Mar.-Apr., May-June 1951.

The study is made up of four parts. Part one deals with the oscillations of a uniform, simply supported beam due to the passage of a smoothly running load. An approximate solution based on the static deflection of a slowly moving load is given. This solution is compared to that given by Stokes. Fair agreement is obtained at low velocities of the moving load. Free oscillations, oscillations due to two smoothly moving loads and impact are considered.

Part two deals with the oscillations of long-span uniform beams whose mass cannot be neglected. The oscillations are caused by a moving uniform load.

Part three treats the damping and resonance of uniform simply supported beams. The last part concerns experimental methods and measurements of bridge oscillations.

In parts one and two, much space is devoted to critical velocities and dynamic coefficients. This is an attempt to consider in a simple manner the different factors, beside span length, that enter into a study of the impact of a beam due to moving load. The study brings together and reviews much of the important earlier work done, primarily by French writers, on this subject.

The papers contain numerous typographical errors and omissions.   Elio D'Appolonia, USA

671. Blakey, F. A., A theory of deflection of reinforced-concrete beams under short-term loads, *Mag. Concr. Res.* no. 7, 3–8, Aug. 1951.

Results of load-deflection tests are reported on tension reinforced-concrete beams with a variety of reinforcement ratios and two different concrete strengths. Results are compared with theoretical determinations. It is assumed that up to first tension cracking the relation $y'' = M/EI_c$ holds, where I_c is the moment of inertia of the transformed section, including concrete in tension. Another expression is used for loads higher than that at which the concrete stress becomes equal to the modulus of rupture f_t. Above that load, concrete in tension is entirely discounted, which leads to $y'' = M/[pbd^3E_s(1 - k)(1 - Ck)]$. This equation can be used for linear (elastic) as well as rectangular (completely plastic) distribution of concrete stresses by appropriate adjustment of k and C.

Experimental load-deflection curves are continuously curved from beginning. Author shows that these are most closely approximated by two straight lines, using the first expression for y'' up to the modulus of rupture load, and the second for higher loads, with k and C for elastic conditions. If k and C for rectangular stress distribution are used, satisfactory agreement is not obtained.

The values of n and f_t were not determined by separate tests on the same concrete, but were assumed to be $30,000/f_{cy}$ and $8(f_{cy})^{1/2}$, respectively, expressions apparently current in Britain and most likely as uncertain as those used in the United States.

George Winter, USA

672. Cowan, H. J., Ultimate strength theory for eccentrically loaded reinforced-concrete columns, *Mag. Concr. Res.* no. 7, 19–22, Aug. 1951.

The theory proposed here is based on a trapezoidal stress distribution at failure, as proposed by F. Von Emperger and developed by V. P. Jensen. The ratio at failure of the plastic component of the concrete strain to the total strain ("plasticity ratio") is assumed to have been previously determined from cylinder tests.

In the case of primary tension failure there are two unknowns to be determined: the location of the neutral axis and the magnitude of the ultimate eccentric load. These are solved for by the equations of statics applied to the forces acting on the cross section.

In the case of primary compression failure there is an additional unknown: the magnitude of the tensile steel stress. The additional equation used is based upon the assumption that plane sections remain plane.

The development of the above theory parallels that employed by C. S. Whitney for his rectangular stress-block assumption. Unlike Whitney, however, author makes no attempt to extend the validity of his equations to the limiting condition of zero eccentricity. Indeed, the case in which the neutral axis lies outside the section is not discussed.

No corroborating empirical data are presented.

Howard Simpson, USA

673. Flügge, W., Statics of the swept wing. 2nd part. Influence on stress distribution of longitudinal variation of loads upon wing section (in French), *ONERA Publ.* 48, 29 pp., 1951.

In a previous paper [AMR **3**, Rev. 251] author considered the swept wing of constant cross section with ribs parallel to plane of symmetry under uniform distortions. Here, an extension is given comprising distortions (torsion, bending, extension, however no shear), increasing linearly along the span. Relations between stresses and the linearly varying moment components and normal force are obtained. No results have been produced for shear loads accompanying the assumed distortions. Shear load requires consideration of stresses produced by relative translation of ribs (shear), which has not been incorporated. Thus, reviewer believes conclusions of this paper are doubtful except for such structures and load systems where, due to symmetry, shear is nullified. Also, the assumed distortions require continuously distributed external loads parallel to the wing axis, approximately

constant per unit of area of wing surface. Author does not discuss the relative importance of this load system. Author arrives at the conclusion that stresses due to linearly varying torque and bending moments are being approximated very well by formulas valid for constant moments.   A. van der Neut, Holland

674. Sibert, H. W., Shear flow in a thin-skin tapered beam, *J. aero. Sci.* **18,** 10, 703–704, Oct. 1951.

Sibert develops a generalization of the equations for the shear flow in the webs of a thin-skinned tapered beam which will yield more accurate results than the usual simple formula $\tau t = q = V \cdot A / I$. He shows that for a specific array of stringers and sheet in a tapered wing, the more exact equation reduces to the usual engineering form.   E. E. Sechler, USA

675. Marcus, H., Load-carrying capacity of dowels at transverse pavement joints, *J. Amer. Concr. Inst.* **23,** 2, 169–181, Oct. 1951.

Tests were made to determine the following: (1) Resistance of concrete to bearing stresses distributed uniformly by dowels of different diameters; (2) effect of dimensions and spacing of dowels; (3) effect of dimensions and strength of concrete; (4) effect of compressibility of subbase; and (5) sliding resistance of dowels coated with various substances.

With respect to (1), tests showed that the bearing stress f_b at rupture load varies—inversely with dowel diameter—from about 5 to 7 times the allowable stress of $0.375 f_c'$, or f_b is roughly 2 times the 28-day cylinder strength f_c'. Furthermore, shallow block test specimens seem to have greater bearing strength than deeper ones. With respect to (2), (3), and (4), loading tests performed on the protruding ends of embedded dowels indicate that: (a) The load at which cracking occurs is but slightly affected by dowel embedment greater than 8 times the dowel diameter; (b) the crack load is almost proportional to concrete depth; (c) on soft subgrades (modulus of compressibility k_s equals 100 to 300 psi), the crack load and the ultimate load are not greatly affected by change in k_s, but the ultimate load of samples on a rigid subgrade may be considerably higher than those on a soft base; (d) a test block width-to-depth ratio greater than 1.5 does not seem to have a definite influence on crack loads. With respect to (5), pull-out tests were made to study the efficiency of various coatings.

Design formulas for allowable loads on dowels are included.
          James P. Michalos, USA

676. Brice, L. P., Bond of steel rods in concrete (in French), *Ann. Inst. tech. Bât. Trav. publics* (*N.S.*) no. 179, 18 pp., Mar.-Apr. 1951.

Paper is a very important contribution to the understanding of the bond phenomenon between steel bars and concrete in reinforced-concrete structures. Using the results of tests conducted by Bichara with electrical strain gages cemented inside hollow bars, author claims that the bond has all the properties of friction. He analyzes several phenomena from this standpoint and gives a formula to determine, for design purposes, the ratio m as a function of the diameter of the bar, its useful bond length, and the percentage of steel. Author claims that this variable m should be used instead of the fixed one, as commonly specified in concrete regulations.

Several discussions are added to the paper. In an appendix, Bichara gives technical details of his tests and points out some complications of the phenomenon when the bar surface is not smooth.   A. J. Durelli, USA

677. Cowan, H. J., The design for ultimate strength of reinforced concrete T-beams, *Civ. Engng. Lond.* **46,** 542, 543; 595–598, 680; Aug., Sept. 1951.

Author reviewed in a previous paper [*Civ. Engng. publ. Works Rev.* **45,** 376–378, 576–578, 723–724] the recent design theories of reinforced concrete and showed the special advantage of the ultimate strength theory and the equivalent rectangular stress block method, based on the works of R. Saliger, K. Hajnal-Konyi, and C. S. Whitney.

In present paper he develops design formulas for T-beams based on the equivalent stress block method, both for balanced design and for under-reinforced sections. The approximate theory neglecting compression in the concrete rib is also discussed. The economy of this type of design and the time saving due to the simplicity of the formulas are demonstrated on four worked examples.   D. Vasarhelyi, USA

678. Pontoppidan, J. A., Carrying capacity of concrete walls without reinforcement (in Danish), *Bygnstat. Medd.* **22,** 2, 36 pp. 1951. Kr. 3.

From report L 10° 50 [*Struct. Res. Lab. roy. tech. College, Copenhagen*] it is observed that concrete walls without reinforcement show ultimate stresses independent of the ratio of slenderness when this ratio (height/thickness $= l/b$) is below 25. Same report deals with the investigation of ultimate stresses at different ratios of eccentricity.

In order to utilize the results given in the report, the actual eccentricities occurring in ordinary structures must be determined. Present paper deals with this problem and treats it by means of the theory of plasticity, presuming that no tension can be transferred to the wall, neither at the upper nor at the lower end. All elastic deformations being neglected, the derivation is valid for $l/b < 25$ only.

The plastic deformations of the concrete will involve such a distribution of the load that the eccentricity e/k will be: $e/k = 3-5\sigma_0/\sigma_T$. k is core radius, σ_0 the ratio of load to area of the wall's cross section, σ_T is the cube strength of the concrete. When $\sigma_0 = 0.2\sigma_T$, tension stresses will occur larger than the tensile strength of the concrete.

Consequently the wall must be treated under the assumption of a horizontal crack having developed somewhere in it. In the crack no tension stresses can be transferred from one part of the wall to the other. The condition of stability is proved to be $\sigma_0 < \sigma_{crit} = 0.6\sigma_T[1 - \theta_m\theta_0 l/(\theta_m + \theta_0 b]$, θ_m and θ_0 being the largest angles of inclination which occur at the ends of the slabs at the top and the bottom of the wall. For ordinary slabs these angles never exceed $2 \cdot 10^{-2}$. The slab is assumed to be supported by the entire cross section of the wall.

As a test, the carrying capacity of walls, tested and described in the report mentioned above, is calculated. A good agreement is found.   From author's summary

679. Johnson, A. Transfer of moments and deformations in concrete beams submitted to long-time loads (in Swedish), *Instn. Byggnadsst. Medd.* **7,** 20 pp., 1950.

Paper is one of a series of studies of problems of theory of reinforced-concrete design. From tests and theoretical analysis, author discusses effect of shrinkage, plastic flow, and crack formation on the distribution of moments and deflections in beams restrained at the supports and subjected to static load for periods up to 500 days.

It is shown that: (1) Plastic flow has little influence on moments but mainly affects deflections. (2) Shrinkage increases support moments both before and after cracking. (3) After cracking, moment at a section is proportional to the tensile reinforcement; but

for usual amount of steel, the change in moment is small for a large change in the steel percentage. A similar conclusion is drawn for deflections. Frank A. Blakey, Australia

680. Wilby, C. B., The strength of reinforced-concrete beams in shear, *Mag. Concr. Res.* no. 7, 23–30, Aug. 1951.

Paper reports partial results of tests to determine ultimate strength of reinforced-concrete beams in shear. Variables include amount of web reinforcement, ratio of shear span to beam depth, and percentage of longitudinal reinforcement. Beams with and without web reinforcement were tested. Results are reported only in graphs and in form of numerous qualitative statements. Reviewer believes, however, that usefulness of results and conclusions are seriously impaired by absence of sufficient data on dimensions of test specimens, manner of loading, properties of materials, and precise statement of range of variables.

C. P. Siess, USA

681. Saal, C., Truck road performance—actual vs. computed, *SAE quart. Trans.* **5**, 1, 18–25, Jan. 1951.

For more than 30 years numerous investigators have attempted to define and evaluate the factors that make up the sum total of resistance to motion of motor vehicles. Author presents a comparison of results of actual road tests with road performance of various trucks as predicted by the method of Stamm and Lamb, a method that may be regarded as the integrated total of the work of many investigators. Tables and equations are presented, and also a method of computation that simplifies, so far as possible, the exceedingly complex and laborious task of predicting road performance.

Author states that actual road tests show agreement within 10% or less between predicted and actual performance, the actual performance being always better than predicted on level roads, and generally less than predicted on grades of 3% or steeper. This method should be of value to designers in estimating the characteristics of new models and types of trucks.

An interesting detail is author's statement: "Investigations are now being made of the possibility of full-scale wind-tunnel tests. . ." In 1923, Professor L. E. Conrad constructed a wind tunnel on the campus of Kansas State College in which actual measurements of air resistance to motion of current models of passenger cars were made that had considerable influence upon the trend in passenger-car body design.

Harold H. Munger, USA

682. Nemesdy, E., Transition curves on main highways (in Hungarian), *Mag. Közlekedés, Mély- és Vizépit. = Comm. civ. Engng. in Hungary* **2**, 9, 36–48, Oct. 1950.

Transition curves must be interpolated, in every instance, between the straight and the curved tracings at road bends, since this is the only way to prevent the occurring centrifugal force from exerting its effect on the vehicles as sudden lateral impacts appearing in the curves. In addition, such tracing corresponds to the natural course of the vehicles, which will thus automatically remain to the right of the road without cutting the curve. The examination extends to the following problems: (a) Shape and minimum transition curvature required by the various types of vehicles; (b) the speed of motor vehicles in the curves; (c) dynamic determination of the minimum length-of-transition curves; (d) suggestions for the numerical determination of the minimum length-of-transition curves; and, finally, (e) the application of long transition curves in connection with curves of great radii. The clothoid curve proved very satisfactory and represents also a favorable solution both from the aesthetic and psychological points of view. Local tracing does not cause any appreciable difficulty in actual practice.

Courtesy Hungarian Technical Abstracts

683. Fritz, B., Compound beam with concrete plate prestressed by tie steel (in German), *Stahlbau* **20**, 8, 97–103, Aug. 1951.

Paper follows another by the same author [*Bautechnik*, 37–42, 1950] which should be read first. The first paper gave an approximate method for calculating the stresses in statically determined compound beams (used in bridges with concrete slab and steel I-beams), by taking into account the shrinkage and creep of the concrete. The theory was based on certain simplifying assumptions concerning the evolution with time of the residual stresses due to shrinkage and creep. The numerical results were in good agreement with those given by a rigorous analysis [cf. Fröhlich, *Bauingenieur*, p. 300, 1949] and, in this reviewer's opinion, they are largely sufficient, owing to the uncertainty affecting the law of creep.

In present paper, author extends the basic calculations to a steel-concrete section, where the concrete is prestressed by steel wires. It is assumed that the concrete slab is rigidly connected with the steel beams and that the concrete is prestressed, both at the same time t_0.

Formulas are given for the stresses at this moment, as well as at the time t_e, where shrinkage and creep have practically stopped. These formulas are the basis for the calculation of statically indeterminate systems such as continuous beams and arches tied by a prestressed concrete floor.

Paper treats completely the case of a symmetrical bridge continuous over three spans and prestressed in the two zones where the bending moments due to the loads are negative. Obtained formulas are applied to the project of a 540-ft bridge over the Ruhr river. Ch. Massonnet, Belgium

684. Parme, A. L., and Paris, G. H., Designing for continuity in prestressed concrete structures, *J. Amer. Concr. Inst.* 23, 1, 45–64, Sept. 1951.

The effect of curved wires or cables in a prestressed beam is evaluated as an equivalent upward load on the beam. No loss of accuracy is involved and the relationships developed are not complex. This approach greatly simplifies the general concept of the action of prestressed beams, since the nonspecialist can visualize the effect of an equivalent upward load more readily than the corresponding effect of a curved prestress cable. The use of ordinary moment-distribution methods for the solution of continuous prestressed beams is made possible by the development of values of fixed-end moments due to prestress. Paper includes sample designs of a simple span, of a fixed-end beam, and of a three-span continuous beam. Reviewer considers this approach a definite simplification that should prove quite helpful.

Phil M. Ferguson, USA

685. Pflüger, A., Buckling of a parabolic arch with a stiff tie (in German), *Stahlbau* **20**, 10, 117–120, Oct. 1951.

The tie, which may be exemplified by the floor of a through bridge, is attached to the arch by hangers. It is assumed to be uniformly loaded and the load from the hangers is assumed uniformly distributed along the arch. The load causing buckling of the entire arch in its plane is sought, taking into account the stiffness of the tie member in bending and of the hangers in tension. Application of energy principles leads to a pair of differential equations, and a solution satisfying one exactly and the other approximately is obtained with the help of the Ritz method. Results show that, contrary to opinions expressed earlier by Mayer-Mita [*Eisenbau* **4**, p. 423, 1913] and Dischinger [*Bauingenieur* 18, p. 508, 1937], buckling of the arch as a whole (1) can be considerably more critical than localized buckling between hangers, and (2) may be significant in the design of arches of slender proportions. A. D. Topping, USA

686. Tör, S. S., Ruzek, J. M., and Stout, R. D., The effect of fabrication processes on steels used in pressure vessels, *Weld. Res. Suppl.* **16**, 9, 446s–449s, Sept. 1951.

A specimen has been developed to allow study of the effect of plate-edge preparation on notch toughness. Flame cutting and especially shearing lower the notch toughness. Postheating at 500 F raises further the transition temperature of flame-cut and sheared specimens. Postheating at 1150 F is noticeably beneficial to the steels.

Postheating at 1600 F lowers the transition temperature below −160 F. This behavior is strong evidence that the adverse effects of flame cutting or shearing are not due to the formation of notches at the surface but, rather, are due to metallurgical changes in the metal at the prepared edge by heating or plastic deformation.

From authors' summary

687. Vedeler, G., Minimum weight of structural parts, *K. norske Vidensk. Selsk. Skr.* no. 1, 23 pp., Sept. 1951.

Paper deals with minimum weight design of structural components of ships, including grillages, portal frames, and deck panels. Design is elastic and based on limiting stress, and is subject to the assumption that supports behave rigidly. Equations are presented for determining spacing of members to give least weight.   Jacques Heyman, England

688. Sidebottom, O. M., Use of the phenomenon of relaxation of stresses to interpret results of tests of railway car wheels, *Proc. Soc. exp. Stress Anal.* 9, 1, 19–26, 1951.

Heating of the tread and rim of a railway car wheel by brake-shoe application and the accompanying temperature gradient may produce stresses in the heated regions above the yield strength of the steel for that temperature. Resulting plastic deformation, which may be considered a relaxation phenomenon, causes redistribution of residual stresses in the wheel when it is cooled. This produces circumferential tensile stresses which may cause fracture. Paper proposes use of the phenomenon of relaxation of stresses at elevated temperatures to predict variables which may influence resistance of wheels to fracture. Theoretical and experimental results indicate residual stresses need not be always detrimental and that reduction in web thickness greatly increases resistance to fracture.   George Gerard, USA

689. Grover, L., MacCutcheon, E. M., Pellini, W. S., Spraragen, W., and Vasta, J., Interpretative report on box girders and small specimens, *Weld. Res. Suppl.* **16**, 7, 321s–326s, July 1951.

Structures made of steels which are not relatively notch sensitive at low temperatures are better able to sustain loads at such low temperatures than those which are made of notch-sensitive steels.   From authors' summary

690. Chronowicz, A., Bending moments due to temperature changes in continuous reinforced concrete structures, *Concr. Constr. Engng.* 46, 8, 241–251, Aug. 1951.

Two cases are considered: Warping of restrained beams or walls due to different temperatures on two opposite faces, and distortions of framed structures due to length changes of members. For warping case, restraint moments are expressed in terms of temperature difference on two faces, coefficients of conductivity and radiation, thickness, and moment of inertia of cracked section of beam or wall as function of thickness and steel percentage. In frames, author shows that thermal length changes of members can be regarded as displacements of joints and resulting fixed-end moments distributed in usual way. Examples are given of moment calculations for both cases, singly and in combination.

C. P. Siess, USA

Rheology (Plastic, Viscoplastic Flow)

(See also Revs. 706, 722, 732, 922, 933)

691. Slibar, A., and Vitovec, F., On the effect of the free surface upon the limit of elasticity of multicrystalline materials (in German), *Federhofer-Girkmann-Festschrift*, Franz Deuticke, Wien, 349–363, 1950.

After reviewing briefly terms defining critical stresses determining the strength of metals, authors point out that, whereas the limits of proportionality and of elasticity in single metal crystals subjected to a state of homogeneous stress coincide with the yield stress, the behavior of polycrystalline metals is characterized by a transition portion in their stress-strain curve. This is due to the fact that in polycrystalline metals not all crystallites can start to deform plastically simultaneously. Their limit of elasticity is caused by the state of residual stresses which remains in them after unloading. In polycrystals the yield stress is determined by interaction between the grains through the mutual resistance exerted by the neighbors of one grain when the latter is ready to deform. In this respect, however, they refer to an effect which has not been considered by former investigators in this field, namely to the behavior of those crystallites which are near to a stress-free surface of a stressed body. Since these are less hindered to deform because they have less neighbors, they respond plastically under locally *lower* values of the mean stress than the grains in the interior of the body. Tests made by the x-ray reflection method seem to corroborate this. It has been found in tensile, compression, and in bending test made with steel and aluminum that the layers nearest to the free surface of specimens start to yield under stresses which are one half or one third as great as the yield stress for uniform yielding. (Here authors quote 17 papers by previous investigators.) They refer also to tests by J. Weerts in 1923—aluminum single and polycrystals—and by A. Kochendörfer ["Plastische Eigenschaften von Kristallen und metallischen 'Stoffen,'" T. Springer, Berlin, 1941] in which it was shown that the stress ordinates in stress-strain curves of a polycrystalline aluminum are twice as great for the same strain as for the single crystal. Then they develop a mathematical theory utilizing the probability calculus and aiming at a determination of a measure of the constraint exerted by the neighbors of one crystallite on this latter, which cannot be reviewed here. (Its principles are not familiar to engineers, apart from the fact that it could not be reproduced in such a brief summary as this review.) They show that near a free surface of a stressed body the normal constraint is reduced. With it goes a decrease of the limit of elasticity within the surface layers. The depth to which the stresses are reduced near the free surface or the depth of this surface effect decreases with increasing plastic strain. Under small permanent strains they estimate it to be of the order of 1 mm.   A. Nadai, USA

692. Holden, A. N., The yielding behavior of iron single crystals, *J. appl. Phys.* 22, 10, 1290–1291, Oct. 1951.

Recent experiments of Holden and Hollomon, Schwartzbart and Low, Cottrell and Churchman [*Trans. AIME* 185, 179, pp. 637, 877, 1949] on yielding of single iron crystals with carbon or nitrogen contents show different results. Experiments of author show: Annealed crystals have no yield point; aging, even after strains in tension as small as 0.45%, at 100 C for 1 hr leads to yielding, yield elongation increasing with increasing amount of carbon; yield phenomena increase with increasing cross section of crystals; crystals aged after rolling show no yield point in subsequent tension tests.

To explain these results, Cottrell's original conception of anchored dislocations by solute carbon atoms is extended in the

following way: According to interaction forces dislocations, first set free at the upper yield point, knock free anchored dislocations when passing the latter within some few atomic distances apart and having velocities approaching velocity of sound, thus causing an avalanche. Calculation of mean free path l between successful collisions (published elsewhere) give 10^{-1} cm for a dislocation density of 10^8 per cm^2, as present in annealed crystals, and 10^{-4} and 10^{-5} cm, respectively, for a density of 10^{11} and 10^{12} per cm^2, respectively, as present in aged crystals. As a consequence yielding does not occur in the first case since l approaches the specimen size, but in the second case as l is considerably smaller. Specimen size effect follows immediately. Missing of yield point in rolled-aged crystals indicates that dislocations in slip planes, operative during rolling, do not affect mean free path for dislocation collisions in slip planes operating during subsequent tension. In reviewer's opinion, this seems to be an unsatisfactory explanation, because identical slip planes may operate in both cases.

F. Wever, Germany

693. Hill, R., On the state of stress in a plastic-rigid body at the yield point, *Phil. Mag.* (7), **42**, 331, 868–875, Aug. 1951.

The yield point of a rigid-plastic body is defined as occurring at that moment when the increasingly applied load causes the initiation of deformation of the body. The load required to cause the yielding of a rigid-plastic body is then compared with the load required to cause the yielding of an elastic-plastic body.

It is shown that the portion of the plastic zone at the yield point, namely, that part where deformation occurs, depends only on the current surface tractions and not on the previous loading program.

By means of the principle of maximum plastic work and its complimentary minimum principle, upper and lower bounds on the yield point are obtained. These limits are then applied to some problems.   Bernard W. Shaffer, USA

694. Gross, B., On creep and relaxation. III, *J. appl. Phys.* **22**, 8, 1035–1039, Aug. 1951.

695. Slibar, A., and Vitovec, F., On the influence zone of strain-hardening (in German), *Schweiz. Arch.* **16**, 3, 80–84, Mar. 1950.

Paper utilizes the theory of probability to determine what proportion of the flow in crystallites surrounding a given grain contributes to its strain-hardening. The effect of any flow in these crystallites on a reference grain depends primarily on their relative orientation with and their distance from this grain. The mutual resistance to flow of two grains grows the larger, the greater their crystallographic orientation deviates.

The contributions to strain-hardening of flow in surrounding crystallites decrease slowly at the beginning as their distance from the grain increases, but converge rapidly to zero after a few grain distances. The summation function representing the collective strain-hardening on a grain is a series that converges rapidly to its limit. The demarcation of the finite influence zone of strain-hardening is formulated herewith.   Dimitri Kececioglu, USA

696. Calnan, E. A., and Clews, C. J. B., The development of deformation textures in metals. II. Body-centered cubic metals, *Phil. Mag.* (7), **42**, 329, 616–635, June 1951.

The qualitative treatment of the development textures in face-centered cubic metals described in Part I has been extended to those of body-centered cubic structure. The main features of tension, compression, rolling and drawing textures have been satisfactorily predicted. The detailed examination of the rolling textures of iron alloys presents some difficulty and necessitates

the introduction of the further assumption that, in adjacent regions within a single metallic grain, there are differences in the ratios of the critical shear stresses. This is justified in that it leads to the explanation of wavy slip lines in α-iron, a well-established fact, and permits the prediction not only of the complete rolling textures of iron alloys but also some of the minor variations among them.

It seems likely that the general concepts of this theory may be applicable to other aspects of the deformation of both single crystals and polycrystalline aggregates. For example, there appears to be a close connection between the alternative directions of rotation discussed here and the development of deformation bands.   From authors' summary

697. Calnan, E. A., and Clews, C. J. B., The development of deformation textures in metals. III. Hexagonal structures, *Phil. Mag.* (7), **42**, 331, 919–931, Aug. 1951.

Hexagonal close-packed metals slip on a single crystallographic plane and not on a system of equivalent planes, as is the case for cubic metals. Due in part to this simple slip mechanism, considerable attention was devoted to the deformation of the hexagonal metals, notably zinc and magnesium, by the German school during the decade 1925–1935 [Schmid and Boas; see AMR 4, Rev. 4145]. There is, however, an additional factor important in the deformation of these metals, namely, the frequent occurrence of deformation twins. The crystallography of the slip and twinning processes has been well established, but there appears to have been no completely systematic treatment on the basis of slip rotations and twinning reorientations. Mathewson and his co-workers have attempted to rationalize deformation textures as a whole, but their method is not applicable to hexagonal metals [see, e.g., Hibbard and Yen, 1948].

The purpose of present paper is to show how the effect of twinning may be introduced into the treatment already developed for face-centered and body-centered cubic metals [Calnan and Clews, AMR 4, Rev. 4152], and how this treatment may be applied to the prediction of the deformation textures in hexagonal metals.   From authors' summary

698. Hall, E. O., The deformation and ageing of mild steel. II, III, *Proc. phys. Soc. Lond.* (*B*) **64**, part 9, 381B, 742–753, Sept. 1951.

Some of the factors which influence the appearance of Lüders bands in mild steel are studied. It is shown that the Lüders band is adequately described by a uniform shear front, spreading over the specimen. In coarse-grained specimens, experiments indicate that this front becomes diffuse; diffuse bands are also present in strain-aged material, but here the diffuse fronts become sharper as the aging becomes progressively longer.

An attempt is made to explain the observed phenomena in the yielding and aging of mild steel in the general terms of a grain-boundary theory. On this hypothesis, a satisfactory explanation of the variation of the lower yield point with grain size may be developed. It is shown that strain-aging must involve two processes: a healing of the grain-boundary films, coupled with a hardening in the grains themselves. A discussion of the possible nature of the grain-boundary film is also undertaken.

From author's summary by T. J. Dolan, USA

699. Wu, M. H. L., General plastic behavior and approximate solutions of rotating disk in strain-hardening range, *NACA TN* 2367, 56 pp., May 1951 = *NACA Rep.* 1021, 1951.

Partly linearized solution is used to generalize author's earlier exact solution [AMR 4, Rev. 2027]. It is based on deformation theory of plasticity applied to rotating disk of initially uniform

thickness, with no central hole and no rim load. Finite strain and actual stress-strain curve are assumed. Comparisons with exact calculation and with bursting tests show very good agreement. Comparison with power-function approximation shows good agreement; with ideal plastic it shows limitations of the latter. General numerical results are presented in tables and curves.

Method can be extended readily to case of central hole and/or rim load. W. Kenneth Bodger, USA

700. **Craggs, J. W., The influence of compressibility in elastic-plastic bending,** *Quart. J. Mech. appl. Math.* **4,** part 2, 241–247, June 1951.

Author attempts to investigate the magnitude of secondary stresses in an elastic-plastic material subjected to bending. Assuming a flow-type stress-strain law and a distortion energy yield condition, author suggests a model which consists of a half-perfect elastic material and a half-plastic incompressible one. In order to eliminate the possible strain discontinuity in such a model, the secondary stresses in the plastic range are determined by employing stress-function concept. A numerical example is given from which author draws the conclusion that forces in the cross-section plane of such a composite specimen under extension may be neglected.

Reviewer believes following points should be pointed out: (a) In Section (2), argument is based on assumption of complete incompressibility in plastic range and determination of secondary stresses is based on assumption of plastic incompressibility only. (b) Reviewer doubts applicability of such a composite model to a beam in which there is no such strain discontinuity existing, as suggested by the model if the cross-section planes remain plane during bending. (c) Secondary stresses do exist, even in elastic bending. (d) There is an obvious misprint in Section (3).

Ling-Wen Hu, USA

701. **Cottrell, A. H., Theory of dislocations,** *Prog. Metal Phys.* **1,** 77–126, 1949.

Author presents a logical review of the present state of development of the theory of dislocations. He begins with a brief historical introduction, followed by basic definitions and concepts, and then progresses to a presentation and discussion of some of the finest details of the subject. The treatment is quite complete and author is careful to present more than one point of view in the several instances where the literature is controversial. Little is said in the review concerning the origin of dislocations, and for good reason, for, as the author so aptly puts it: "The excuse for delaying the discussion of the origin of dislocations until the last is that this subject is at present the most obscure part of the theory of dislocations." M. C. Shaw, USA

702. **Nabarro, F. R. N., The interaction of screw dislocations and sound waves,** *Proc. roy. Soc. Lond.* (*A*) **209,** 1097, 278–290, Oct. 1951.

The model of a dislocation given by Peierls [*Proc. phys. Soc.* **52,** p. 34, 1940] is adapted to dynamical problems by considering inertia of matter in continuum between the two sheets of atoms defining a glide plane. Scattering of long shear waves by a screw dislocation is studied. Shear waves obliquely as well as normally incident on a dislocation are considered. An equation of motion of a dislocation is given. Using the analogy of an electron in an electromagnetic field, the force on a dislocation in the direction of wave propagation is determined. These results are applied in the second part of the paper to an analysis of the resistance to motion of a dislocation caused by its interaction with lattice vibrations as determined by Leibfried [*Z. Phys.* **127,** p. 344,

1950]. The conclusion is that Leibfried's resistance is qualitatively correct but too large. W. H. Hoppman, II, USA

703. **Witte, R. S., and Anthony, R. L., Stress-temperature studies of transitions in rubbers,** *J. appl. Phys.* **22,** 6, 689–695, June 1951.

The apparent "second-order" transition temperatures of natural gum, butyl gum, loaded GR-S, and loaded Hycar OS-10 rubbers were studied by a "stress-temperature" technique. Tensile specimens were stretched at room temperature to a predetermined elongation and held there until the stress relaxed to a relatively constant value (from 6 to 24 hr). The temperature was then lowered in steps (at an unspecified rate) and the stress plotted as a function of temperature. Stress decreased linearly with decreasing temperature until the transition region was reached. At this point, the slope of the curve changed abruptly, and further lowering of the temperature resulted in sharp increases in stress. Stress-relaxation tests above and below the transition showed no stress decay above the transition but pronounced relaxation below this temperature. The transition region of butyl and Hycar OS-10 was found to decrease with increasing initial elongation, but that of natural rubber was independent of elongation. The transition of natural rubber was -61, C at 10, 50, or 150% elongation; butyl gum stock was -49 C at 33% and -65 C at 533%; Hycar OS-10 was -15 C at 75%, but indeterminate, perhaps $+5$ C, at 10%. Transition temperatures determined by volumetric methods were somewhat lower than those obtained by stress-temperature studies.

R. H. Carey, USA

Failure, Mechanics of Solid State

(See also Revs. 672, 699, 719, 720, 737, 740)

704. **Tízhnova, N. V., On the question of the mechanism of fatigue failure of steel** (in Russian), *Zh. tekh. Fiz.* **21,** 2, 187–195, Feb. 1951.

Effects of fatigue on "Steel 20" with 31,000 psi endurance limit were made by counting the decreasing number of spots in x-ray patterns. As Barrett also reported ["Structure of metals," p. 379], spots become more diffuse with increasing numbers of cycles and with higher stresses, but no particular change occurs when the endurance limit is crossed. F. A. McClintock, USA

705. **Breen, J. E., Effect of amplitude of dynamic stress on high temperature fatigue life,** *ASTM Bull.* no. 177, 38–39, Oct. 1951.

Generally, testing of materials at elevated temperatures has centered around stress rupture and creep testing. However, as dynamic stresses occur in many high-temperature applications, the effect of superimposing a cyclic stress on a static test seemed worth investigation. After citing the work already done by others, author states that his own experiments were meant to provide more detailed information about this effect for a single-phase material (α-brass, annealed at 1000 F for 16 hr) at a temperature of 550 F. Two static stresses (9000 and 12,000 psi) were applied. Relaxation of the loads by creep was avoided by using a Sonntag direct stress fatigue-testing machine, model SF-4. Dynamic stress was varied from zero up to approximately 100% of the static stress. For both static loads the rupture life was found to be progressively decreased when the amplitude of the dynamic stress was raised. With a dynamic stress of 9000 psi the rupture life was decreased to only approximately 1% of the time to rupture for zero dynamic stress. With a dynamic stress of 5000 psi the time to rupture was decreased much less,

namely to approximately 25% of the original value. The elongation was found to be improved by increasing the strain rate (by choosing a higher value of either the static or the dynamic stress).

J. H. van der Veen, Holland

706. Torre, C., On the physical significance of Mohr's envelope (in German), *ZAMM* **31**, 8/9, 275–277, Aug./Sept. 1951.

Author discusses limiting failure curves of the type $\tau = \tau(\sigma)$ where τ and σ are normal and shear stresses, respectively, acting in plane of failure. The usual Mohr envelope $\tau_A = \tau_A(\sigma)$ is obtained from the relation $\sigma_1 = \sigma_1(\sigma_3)$ where σ_1 and σ_3 are maximum and minimum principal stresses, respectively, at failure. Author shows that real contact between stress circle and Mohr envelope curve is obtained only if $\sigma_1' \geqq 0$ where $\sigma_1' = d\sigma_1/d\sigma_3$. Another limiting curve $\tau_w = \tau_w(\sigma)$ is obtained directly from the stresses σ_1 and σ_3 and the fracture angle α observed in tests; experimental investigations are cited to show that $\tau_A(\sigma)$ and $\tau_w(\sigma)$ are not identical, although for certain cases differences between these curves may be small. Discussion of reasons for differences between observed and theoretical fracture angles is also given on the basis of a limiting failure surface $F(\sigma_1\sigma_2\sigma_3) = 0$.   A. M. Wahl, USA

707. Brown, W. F., Jr., and Sachs, G., A critical review of notch sensitivity in stress-rupture tests, *NACA TN* 2433, 29 pp., Aug. 1951.

A review of English and German literature on notch stress-rupture testing revealed several facts regarding the general influence of notching on elevated-temperature long-time properties. (1) Notch sensitivity was observed in several alloys including a group of heat-resisting steels; (2) the sensitivity appears related to ductility—highly ductile metals were not notch sensitive, whereas increasing notch sensitivity developed if the ductility in unnotched specimens fell below 5%; (3) in heat-resisting low-alloy steels, nickel in amounts greater than 0.7% appears to increase notch sensitivity greatly; (4) notch sensitivity in Cr-Ni-Mo low-alloy steels appears to be associated with a time-and-temperature-dependent precipitation; and (5) the influence of notch sharpness is shown to be the same in stress-rupture tests as in room-temperature notch tensile tests.

From authors' summary by Carl E. Hartbower, USA

Material Test Techniques

(See also Revs. 724, 726, 740)

708. Deisinger, W., Testing of heat resistant materials (in German), *Schweiz. Arch.* **17**, 10, 299–305, Oct. 1951.

Author points out difficulties inherent in various kinds of tests, physical and chemical, particularly when basic properties of material are sought, unaffected by test conditions. He concludes that the most effort should be concentrated on long-time creep or stress-to-rupture tests, especially since these are carried out in practically the same manner everywhere, and that increase of basic knowledge by accumulation and evaluation of data from such tests should be comparatively easy. To save time and expense author proposes set-up in which several bars can be simultaneously tested, all being subjected to same temperature, but with provision for individual loading and measurement of extension.

C. W. Smith, USA

709. Hastings, C. H., and Carter, S. W., Inspection, processing, and manufacturing control of metals by ultrasonic methods, Symp. Ultrasonic Test., *ASTM Spec. tech. Publ.* 101, 14–61, 1951.

This report summarizes available technical literature through 1946 on the subject of ultrasonics as applied to metals. Particular emphasis has been placed on the detection of flaws or discontinuities in metals since the bulk of the literature encountered has dealt with this application. However, other metallurgical applications were encountered and have been included for the sake of completeness and to promote speculation along these lines.   From authors' summary

710. Dietz, A. G. H., Closmann, P. J., Kavanagh, G. M., and Rossen, J. N., The measurement of dynamic modulus in adhesive joints at ultrasonic frequencies, Symp. Ultrasonic Test., *ASTM Spec. tech. Publ.* 101, 117–129, 1951.

See AMR 4, Rev. 4463.

711. Kepes, A., A new apparatus for measuring of the modulus of elasticity and internal friction of solids (in French), *Instrum. Measur. Conf., Stockholm, Trans.*, 136–138, 1949.

A simple dynamic method for measuring moduli in the range from 0 to 2000 rpm is presented. A cylindrical sample is used, the base of which is rotated in a rigid holder. A perpendicular force is applied at the other end of the cylinder so that the elements of the cylinder are subjected to sinusoidal stress variations. Curves of internal friction coefficient vs. frequency are given for three materials: ebonite, saran, and polyvinyl chloride. Method has the advantage that the frequency is continuously variable, unlike resonance methods, but requires fairly precise observations.

Donald G. Ivey, Canada

712. Jellinghaus, W., and Möller, H., Application of nondestructive testing in the roll mill (in German), *Stahl u. Eisen* 71, 19, 995–1002, Sept. 1951.

This is an extensive rather than an intensive report on special methods of testing used in the rolling mills of Germany, and it indicates that few good possibilities are being overlooked. Included are several hints of new indicators of physical characteristics which are being investigated for future use. Nondestructive testing offers such great advantages over the ordinary type of sampling tests in thoroughness and in making complete use of material that article should interest many in other fields.

The agencies used to give indications of variations in quality include heat, light, x rays, electron streams (beta rays), sound, ultrasonic transmission, magnetism, and electric currents. Twenty-four references are given and sixteen figures show test equipment and its operation.   W. C. Johnson, USA

713. Van Dongen, L., Determination of residual stresses (in Dutch), *Ingenieur* **63**, 38, 0.45–0.51, Sept. 1951.

Systematic study of the accuracy of the hole-drilling method for the determination of residual stresses by means of electrical wire gages. Formulas established by Soete are corrected by taking into account transverse sensitivity of the gage. Author used A5 Baldwin gages; diameter of hole was 6 mm; distance between hole and gage, 2 mm. For stresses up to 15,600 psi in mild steel, method is reliable within 10%. Systematic error is probably due to triaxiality in the neighborhood of the hole.

W. Soete, Belgium

714. Nolle, A. W., and Westervelt, P. J., A resonant bar method for determining the elastic properties of thin lamina, *J. appl. Phys.* 21, 4, 304–306, Apr. 1950.

The elastic properties of a thin layer of cement material between two cylindrical steel bars are determined from the resonance-vibration behavior of the cemented bar and a similar continuous steel bar. Young's modulus of the cement is determined

from the change in resonance frequency of the bar with and without the cement. The damping or imaginary part of the modulus is determined from the band width of the resonance peak. An error of about 10% can be expected in the elastic values. The method is applicable to the determination of the elastic properties of thin lamina and to the evaluation of adhesives.

Lawrence Nielsen, USA

715. Hoeffgen, H., and Back, G., Comparison of methods of nondestructive testing of concrete with ball testing instruments (in German), *Bauingenieur* **26**, 10, 297–300, Oct. 1951.

Author discusses different methods of nondestructive testing of concrete, techniques similar to Brinell-hardness measurements. Three instruments were tested, pendulum hammer, hand-operated hammer, and spring-fired hammer; the penetrating ball for the first two is 25-mm diam and for the last, 10-mm diam. The mechanical and physical limitations on the use of each apparatus are discussed.

Use of these methods of test depends on empirical relationship between surface penetration of the ball and the compressive strength of the concrete. Author discusses effect of numerous factors, such as grading of aggregate, compaction of concrete, age, and moisture content in this empirical relationship.

Conclusions are drawn from results of tests, details of which are not given. Conclusions are that, for ordinary concrete at an age of 28 days, methods are fairly reliable but can be grossly in error for very high and very low strength material, and for concretes in which coarse aggregate is large and the amount of mortar is small.   Frank A. Blakey, Australia

716. Grassam, N. S. J., and Fisher, D., Tests on concrete, with electrical-resistance strain gauges, *Engineering* **172**, 4469, 356–358, Sept. 1951.

A report on technique developed by authors for using electrical-resistance strain gages on the surface of dry concrete. Reliability of the technique is demonstrated by reference to tests on concrete tension and compression specimens. From each test electrical strain indicated by the subject gages is compared with strain indicated by mechanical strain gages. Comparisons show the strain-sensitivity factors of the electrical-resistance strain gages to be essentially constant and equal to the factors obtained on steel specimens. Consistent readings were obtained and strains as small as 1×10^{-6} were easily detected.

M. J. Holley, Jr., USA

Mechanical Properties of Specific Materials

(See also Revs. 663, 698, 703, 704, 707, 711, 714, 716, 904)

717. Zoja, R., On the alternate torsion test on wires (in Italian), *Metallurgia ital.* **63**, 10, 421–424, Oct. 1951.

In the alternate torsion test on wires three distinct periods are generally identified: Uniform deformation along the useful length; localized deformation on a short distance; and rupture with formation of the characteristic more or less closed pencil of fibers.

The results obtained from the alternate torsion-stress tests carried out not only on steel but also on copper, nickel, aluminum and their alloys, permit this test, which has a technological character, to be classified as the best auxiliary test for the tensile test of wires.   From author's summary

718. Jenkins, W. D., and Digges, T. G., Creep of annealed and cold-drawn high-purity copper, *J. Res. nat. Bur. Stands.* **47**, 4, 272–287, Oct. 1951.

A study was made of the effect of temperature and stress on the creep behavior at 110, 250, and 300 F of high-purity oxygen-free high-conductivity copper initially as annealed and as cold-drawn 40% reduction of area. The resistance to creep and to fracture at all these test temperatures was increased by cold-drawing the copper prior to testing in creep; however, this superiority in creep properties was accompanied by a decrease in ductility and time to rupture. Dissociation of parent grains into crystals of microscopic dimensions and the presence of strain markings were evidenced in all specimens carried to complete fracture in creep. Correlations are made of these changes in structure with time, stress, temperature, and the discontinuous flow, as evidenced by the creep rate-extension curves.   From authors' summary

719. Chudakov, E. A., and Uzhik, G. V., On a special resistance of metals in repeated compression (without contact stresses) (in Russian), *Dokladi Akad. Nauk SSSR (N.S.)* **76**, 1, 37–40, Jan. 1951.

Few results were known on behavior of metals under repeated compression and no information was published on the effect of stress concentration in such tests. Tests in repeated bending and compression on steel specimens were, therefore, undertaken in a 5 (metric)-ton hydraulic pulsating machine. The bending load on beams with quadratic, trapezoid and unsymmetric I-section was transmitted indirectly. Some of specimens possessed a sharp transversal groove on the compression side or on both parts of the beam. The average stress in bending tests was a little above the yield stress (25 tons/sq in.) of material. All bending specimens failed on the tension side, although stresses in grooved compression side were 2 to 5 times higher. Tests in fluctuating compression with stresses between 11.3 and 47.8 tons/sq in. showed no signs of failure. When compressed between 7 and 52.8 tons/sq in., specimens showed cracks after a few thousand cycles, but no failure occurred after 20.10^6 cycles.

Anton Kuhelj, Yugoslavia

720. Roš, M., Fatigue of metals. Bodies with notches. Results of tests in the L.F.E.M. (1948–1950) (in French), *Rev. Métall.* **48**, 10, 723–733, Oct. 1951.

Paper is summary, mainly in form of graphs, of research results on fatigue obtained at Swiss Federal Institute for Testing Materials, Zürich—of which author was head until retirement two years ago—and which were published in full as 160-page Rep. 173 of that Institute in 1950.

Fatigue tests with tubular specimens of mild steel, cast steel, weld metal, aluminum alloy under internal pulsating pressure and superimposed synchronized tension or compression show agreement with what author calls Coulomb-Mohr hypothesis of failure. Pure shear fatigue strength is 70 to 80% of fatigue strength in pure tension. Gough's researches in England (Pres. address, *Proc. Instn. mech. Engrs.*, 1948) under combined tension and bending had shown shear strain-energy criterion to be satisfied for fatigue of ductile materials, so that pure shear fatigue strength should not exceed 60% of tensile fatigue strength.

Tests with perforated and notched test pieces showed that fatigue strength is almost constant for range of stress-concentration factors between 4 and infinity. Large decrease in fatigue-strength results for stress-concentration factors increasing from 1 to 4. Compressive fatigue strength of unnotched mild-steel specimens is 60% greater than tensile fatigue strength. Same increase for notched specimens is only 10 to 20%. Author found no evidence for influence of stress gradient.

R. Weck, England

721. Narayanamurti, D., and Jain, N. C., The damping capacity of some Indian timbers: II—Logarithmic decrement in flexure, *J. aero. Soc. India* **3**, 3, 79–87, Aug. 1951.

In continuation of previous tests on the damping capacity of Indian woods in torsion, data are presented on the damping capacity and moduli of elasticity of air-dry Indian timbers in flexure parallel and perpendicular to grain, on the basis of photographic records of light beams reflected from the oscillating test specimens of $12 \times 1 \times 1/8$ and $6 \times 1 \times 1/8$-in. sizes, respectively.

The logarithmic decrement along the grain of the tested woods varied from 0.01008 to 0.02307. The ratio of the decrement across to that along the grain varied from 2.07 to 3.93 and 6.44, with an average of about 3.5. The logarithmic decrement in torsion was found to be about 1.5 to 4.5 times that in flexure.

E. George Stern, USA

722. Kingston, R. S. T., and Armstrong, L. D., Creep in initially green wooden beams, *Austral. J. appl. Sci.* **2**, 2, 306–325, June 1951.

Paper describes an investigation of the reaction of simple beams with $3^1/4$-in. square cross sections and varying spans to continuous and intermittent loads, most of which were applied at quarter points. Stresses in extreme fibers varying from 250 to 6000 psi were developed in initially green mountain ash (*Eucalyptus regnans* F.v.M.) with the average modulus of rupture in a green condition of 9000 psi. Deflections of beams stressed to 250 psi and over increased from three- to sixfold and then approached constancy within a period of one year provided that failure did not occur. When failure started, deflection continued at an increased rate. It was indicated that temperature may appreciably influence creep. All beams stressed to 6000 psi failed within five months, one-quarter of those stressed to 4000 psi failed within 10 months, and one-sixth of those stressed to 2000 psi failed within 15 months. Residual deflection was found to be greater for intermittently loaded beams of the shorter spans, suggesting the occurrence of creep in shear. Deflection-time curves for periodically loaded beams were similar in general shape to those for continuously loaded ones.

Stephen B. Preston, USA

723. Klinger, R. F., Tensile properties of some aircraft structural materials at various rates of loading, *Proc. ASTM* **50**, 1035–1050, 1950.

Paper reports on the effect of rate of loading in tension on aircraft structural materials. Materials tested are 24S-T and 75S-T aluminum alloys, polymethylmethacrylate, and some laminated plastics. All tests were carried out at room temperature. Tests run at slow and medium rates of loading are rated by the amount of time required to reach yield strength and ultimate strength. For very high rates of loading, an impact testing machine was used and the times involved could be estimated only approximately.

It was found that the initial modulus of elasticity, yield strength, and ultimate strength of the aluminum alloys were not influenced very much by the states of loading; however, in the case of the plastic materials, all three properties showed a marked increase with increase in rate of loading.

For the plastic materials, the results obtained agree generally with data obtained by Hsiao and Sauer for polystyrene [see AMR 4, Rev. 3255], and Findley for cellulose acetate [*Proc. ASTM* **41**, p. 1231, 1941]. However, the tests in those cases were carried out at constant rates of strain, rather than at constant rates of loading.

Yoh-Han Pao, USA

724. Painter, C. W., The measurement of the dynamic modulus of elastomers by a vector subtraction method, *ASTM Bull.* no. 177, 45–47, Oct. 1951.

An apparatus is described in which a double shear test specimen driven by a mechanically oscillated platen executes simple harmonic motion. The total force transmitted through the specimen is measured using a wire strain-gage force pickup coupled to a vacuum tube voltmeter. Then voltage signals from the force pickup and from a displacement pickup similar to the former are combined in an ingenious manner so that the resultant voltage yields the damping component of force. The amplitude of motion is measured with the displacement pickup or by other means. The real and imaginary parts of the dynamic modulus of elasticity are calculated.

B. M. Axilrod, USA

725. Hughes, D. S., Blankenship, E. B., and Mims, R. L., Variation of elastic moduli and wave velocity with pressure and temperature in plastics, *J. appl. Phys.* **21**, 4, 294–297, Apr. 1950.

The variations in velocity of dilatational waves in the pressure range 0 to 15,000 psi and temperature range 30 to 90 C have been measured for samples of polystyrene, Lucite, and polyethylene. The velocity of rotational waves was also measured for polystyrene and Lucite. In polyethylene no trace of a rotational wave could be identified. The elastic moduli and Poisson's ratio are computed over the experimental range.

From authors' summary

726. Frankel, L. P., and Radcliffe, C. W., Bearing strength of laminated plastics, *ASTM Bull.* no. 175, 71–75, July 1951.

Paper describes a test for obtaining the bearing strength of $1/8$-in. plastic-sheet material. In contrast to the continuous stress-strain test, this repeated load test consists of loading the specimen to a certain load (100 lb), reading the deformation at that load, releasing the load, and measuring the permanent set. The specimen is then reloaded to 200 lb and the procedure is repeated. This is continued until a rapid increase in the permanent set is observed. For most of the plastics tested there is some definite break in the stress permanent set curve. The stress at which this occurs is taken to be the yield strength.

Authors claim this approach is less arbitrary than the usual procedure of applying the 4% deformation criterion to the stress-strain curve obtained by continuous load methods. The repeated load test also gives the amount of permanent set to be expected. The strength values obtained by the two different types of test agree closely.

Yoh-Han Pao, USA

727. Meredith, R., Cotton fibre tensile strength and x-ray orientation, *J. Text. Inst. Trans.* **42**, 7/8, T291–T299, July/Aug. 1951.

728. Backer, S., The relationship between the structural geometry of a textile fabric and its physical properties. II. The mechanics of fabric abrasion, *Text. Res. J.* **21**, 7, 453–468, July 1951.

A general mechanism of cloth abrasion is proposed—one which differs from the process of surface attrition of solid bodies because of the complex geometry of fabric surfaces and the viscoelastic properties of textile fibers. The total mechanism is comprised of three elements, the relative magnitudes of which depend upon the nature of the abradant, the behavior of the fiber in the fabric structure, and the general conditions of rubbing. The three elements of abrasion are friction, surface cutting, and fiber plucking. Friction and surface cutting cause direct damage to the fiber at local points of contact with abrasive particles. Plucking may cause immediate or dynamic fatigue rupture of the fiber at that point along the fiber length where maximum stress concentration is built up. The horizontal force developed in friction or surface cutting will also develop components along the fiber axis and thus

lead indirectly to tensile or bending rupture which normally results from fiber plucking.

The nature of the frictional phenomenon which takes place when two solids are rubbed together is discussed. The relationships among forces developed in the cutting of metal is described, and the utility of photoelastic techniques in the study of surface forces in high polymers is pointed out. The forces built up along the fiber axis during fabric abrasion are considered in the light of the reported dynamic fatigue and flexural endurance of fibrous high polymers. While flexing fatigue and tensile recovery properties form a striking parallel to the general abrasion resistance of textile fibers, conditions of rubbing are shown to nullify and even reverse expected differences between abrasive durabilities of fibers used in experimental cloths.

From author's summary by Rogers B. Finch, USA

729. Brosch, C. D., Interpretation of tests on resistance of molding sands (in Portuguese), *Inst. Pesq. Tecnol. São Paulo, Publ. 431 = Bolet. Assoc. Brasil Metais* **7**, 23, 193–206, 1951.

A series of tests with various molding sands are described and interpreted as they are used in foundries (green-sand molds, dry-sand molds and skin-dried molds). Two types of apparatus used, pendulum and coil, are described in detail. Tests refer primarily to compression, shear, and tension, and influence of various constituents and admixtures on these properties is investigated. Also, the effect of molding conditions is discussed. Author concludes that the strength values are more indicative for dry sand and skin-dried (baked) sand than for green-molding sand.

J. J. Polivka, USA

730. Smith, G. V., Seens, W. B., and Dulis, E. J., Hardened alloy steel for service up to 700 F, *Proc. ASTM* **50**, 882–892, 1950.

Paper describes properties of two steels, SAE 4340 and 0.40 carbon Ni-Cr-Mo-V steel, quenched and tempered to a hardness of Rockwell C43. Tests were conducted in tension at 75, 500, 600, and 700 F, in compression at 75 F, for creep-ruptured strength at 500, 600, and 700 F, and for Charpy impact strength within the temperature range −315 to 75 F.

William J. Anderson, USA

731. Siefert, A. V., and Worrell, F. T., The role of tetragonal twins in the internal friction of copper manganese alloys, *J. appl. Phys.* **22**, 10, 1257–1259, Oct. 1951.

Tests on small electromagnetically excited flexure specimens reveal that, for alloy 88%-Mn 12%-Cu quenched from 925 C, internal friction decreases after each stage of anneal. This is attributed to reduction of tetragonal twins.

Alexander Yorgiadis, USA

732. Du Toit Meyer, M. A., A theory of Rockwell ball hardness, *Appl. sci. Res.* **A3**, 1, 11–26, 1951.

Rockwell hardness is defined in terms of the differential penetration of a ball into a surface under two different loads. It can, therefore, be immediately related to ball penetration under a single load (Meyer law). Author obtains this relation between Rockwell and Meyer hardness, and checks it experimentally. Theoretical formulas for the elastic recovery following load removal are also proposed.

Rodney Hill, England

733. Weil, L., Variation of Young's modulus of a ferrite possessing density (in French), *Ann. Inst. Fourier Univ. Grenoble* **2**, 207–213, 1951.

Using a ring of a ferrite material $Fe_2O_3Ni_{0.3}Zn_{0.7}O$, a magnetostrictive vibration is generated in the ring by using a steady polarizing field and an alternating driving voltage. The magneto-

strictive expansion is only 1.3×10^{-6} so that only a weak vibration can be excited. Near the resonance of the ring, the measured Q becomes very small due to loss of energy to the mechanical vibration, and the resonance frequency of the ring can be accurately located. From the ordinary ring formula $f_0 = (E/\rho)^{1/2}(1/2\pi r)$ and the density ρ, the value of Young's modulus can be determined. Ring densities from 3.88 to 5.06 were investigated, and it was shown that there was a corresponding variation in the value of Young's modulus from 8.24×10^{11} dynes/cm² up to 19.8×10^{11} dynes/cm². This latter figure approaches the value for iron.

Warren P. Mason, USA

734. Bozorth, R. M., Mason, W. P., and McSkimin, H. J., Frequency dependence of elastic constants and losses in nickel, *Bell Syst. tech. J.* **30**, 4, part I, 970–989, Oct. 1951.

This interesting paper describes the determination of the three elastic constants of monocrystalline nickel, using pulses of elastic waves of frequency 10 mc/s and of duration 0.001 sec; the specimens were short cylinders, about 0.5 cm long and 2.5-cm diam. and the measurements were carried out in magnetic fields (H) varying between 0 and 6000 oersteds. With increasing H, the value of Young's modulus E is known to increase (ΔE effect). For polycrystalline nickel at lower frequencies (10–50 kc/s), ΔE is about 15–30%, when H increases from zero to a value sufficient to cause saturation; in the present experiments, the corresponding value of ΔE was only about 3%. This difference in order of magnitude was attributed to micro-eddy-current damping which gives rise, at high frequencies, to relaxation of the motion of the walls of the magnetic domains; the change in the magnetization of a domain is visualized as giving rise to eddy currents in and around it, resulting in a loss of energy which depends, among other factors, on frequency and on the shape and size of the domains. This hypothesis was verified by measuring the ΔE effect and the decrement of polycrystalline rods of nickel, using lower frequencies (5 − 200 kc/s). From measurements on the (decrement, frequency) curve, the size of the average domain was deduced to be about 0.04 mm, agreeing with optical measurements which give values lying between 0.01 and 0.2 mm.

R. M. Davies, Wales

735. Krishnamurti, D., Evaluation of the elastic constants of diamond from its Raman frequencies, *Proc. Indian Acad. Sci.* (*A*) **33**, 6, 325–332, June 1951.

Expressions for the elastic constants of diamond have been derived in terms of the force constants calculated by Ramanathan which express the interaction of any atom with its 28 neighbors. The calculated values are: $C_{11} = 9.6 \times 10^{12}$; $C_{12} = 3.9 \times 10^{12}$; $C_{44} = 4.2 \times 10^{12}$ dynes/cm², while the values experimentally determined by Bhagavantam and Bhimasenachar are: $C_{11} = 9.5 \times 10^{12}$; $C_{12} = 3.9 \times 10^{12}$; $C_{44} = 4.3 \times 10^{12}$ dynes/cm².

From author's summary

736. Swan, D., Lytle, A. R., and McKinsey, C. R., Stress relieving and fracture strength, *Weld. Res. Suppl.* **16**, 3, 135s–145s, Mar. 1951.

The factors of straining and heating involved in the application of the controlled low-temperature stress-relieving process have been investigated by studying their effect on transition temperature. From the results of these tests and a series of mechanical tests, it was concluded that there was no appreciable metallurgical effect from the heating and straining accompanying this process.

It has been proposed that residual stresses may be quite important in the failure of welded structures under conditions which inhibit plastic flow. The amount of plastic deformation occurring during a destructive test, as measured by the amount of strain

hardening which takes place at the initiation of fracture, was shown to be dependent upon severity of testing conditions. Available data were analyzed, and conditions necessary for the study of the effect of residual stresses were outlined. A few preliminary tests were made, and the results were compatible with the hypothesis, although further tests are necessary to confirm its validity.   From authors' summary

737. Dimmick, H. M., and McCormick, J. M., Variation, with temperature, of the cracking velocity, *J. Amer. ceram. Soc.* **34, 8, 240–241, Aug. 1951.**

Experiments revealed a small, but real decrease in the limiting crack velocity as the temperature of a soda-lime glass was raised. The mean velocities decreased from 1510 m/sec at 80 K to 1455 m/sec at 1455 K. Electronic measurements were made of the time required for a crack to break two parallel platinum lines fired into the surface of glass specimens. Care was taken to have the gage section at a sufficient distance from the crack origin to insure that the crack speed had stabilized before measurement.

No theory is offered as to the cause of this temperature effect.
Winston Duckworth, USA

738. Van Zee, A. F., and Babcock, C. L., A method for the measurement of thermal diffusivity of molten glass, *J. Amer. ceram. Soc.* **34, 8, 244–250, Aug. 1951.**

Measurements of thermal diffusivity were made on molten glass in a cylindrical crucible, heated by external electrical heating elements. Method consisted in superimposing a small sinusoidal temperature variation on the glass at its outer radius, with a total period of 2 or 4 hr, and measuring the angular time lag of the temperature at the center behind that at the outer radius from continuous temperature charts. Equations given show that diffusivity is directly proportional to this angular lag. For a soda-lime glass, it was found that diffusivity increased exponentially with temperature from 0.016 cm^1/sec at 700 C to 0.21 at 1400 C.   E. B. Shand, USA

739. Peukert, H., Mechanical and optical behavior of warm-stretched Plexiglas M33 in tension (in German), *ZVDI* **93, 26, 831–835, Sept. 1951.**

The stress-optical coefficient of Plexiglas proves to be a nearly linear function of the percentage of warm-stretching. Material has a marked tendency to cold-flow under constant loading; magnitude of cold-flow depends on the amount of warm-stretching and has a maximum in the neighborhood of 95%. The optical coefficient depends on the strain as well as the stress; its value is determined from the cold-flow lines.

Author suggests using the value of the optical coefficient to determine the amount of warm-stretching with thermoplastic, and the degree of polymerization with thermosetting resins.
R. G. Boiten, Holland

740. Thompson, P. F., Corrosion of metals. Metals under stress. I. Aluminum, *Aero. Res. consult. Comm. Austral. Rep.* **ACA-49, 22 pp., July 1950.**

Paper presents results of study of stress corrosion of commercially pure aluminum (Al2S) in annealed and hardened condition. Quantitative aspects of corrosion were measured by determining the solution potential of aluminum under various stresses when immersed in different corrosive media. Potential measurements were taken during aeration and de-aeration of the solutions. Descriptions and illustrations of apparatus are given. Author concludes that stress does not appreciably affect the solution potential of a metal, although it may produce strains that cause a breakdown of protective films, thereby exposing fresh active metal surfaces so that corrosion is accelerated. In cavi-

ties, arising from shock or concentrated stresses or from chemical solution of segregated impurities, and in the absence of oxygen, hydrogen evolution causes deep pitting and tunneling which may continue indefinitely.

Reviewer believes that stress concentration and accompanying concentration of strains at the root of the crack or pit may cause continued breakdown of protective films in this region as fast as films can form. Thus, cracks once formed are self-propagating. Growth of cracks may also be accelerated because the oxygen supply is depleted at the root of the crack and an oxygen concentration cell is thereby established.   Carl A. Keyser, USA

741. Robinson, G. I., Influences of grain flow on the strength of lugs, *Aircr. Engng.* **23, 271, 257–260, Sept. 1951.**

Abnormal types of failures, namely through the sides instead of through the ends of the lugs and sometimes seriously (75%) premature, were encountered in tests with transverse-grain lugs, machined from DTD 683 and DTD 364 forgings, although material properties, even across grain, were satisfactory (elongations across grain 6.1 and 8.2%, respectively). Lugs failed at a section where there was 22–29% theoretical excess of material, and stress-concentration factors as high as 1.4 (instead of the normally expected 1.1) must have occurred in the sides. As the more usual effects, viz., low transverse-grain strength and low elongations across grain, were absent, the hypothesis was put forward that the phenomena should be related to the (actually observed) difference between the tangent moduli E_t. In the beginning of the plastic region, E_t along the grain is appreciably less than E_t across the grain.

This would give rise to modifications in the bending-moment distribution around the eye of the lug, by which the points of contraflexure on the sides of the lug will have moved downward, so that final bending moments through the sides may be considerably in excess of normal bending moments. Using this hypothesis, behavior was predicted of lugs having various geometries, and predictions were satisfactorily checked by preliminary experiments. Pending further test work, author suggests the use of a 50% margin of strength on sections through the sides of transverse-grain lugs and a margin of 10% on sections through the ends of longitudinal-grain lugs.

J. H. van der Veen, Holland

742. Richards, J. T., and Smith, E. M., The forming characteristics of beryllium copper strip, *Proc. ASTM* **50, 1085–1099, 1950.**

Results are reported of tests to correlate the forming characteristics of beryllium-copper strip prior to precipitation hardening with usual specification properties including tensile strength, elongation, and Erichsen cup test. Formability is expressed as the minimum safe radius for forming a 90-deg bend by means of a punch and die set-up. Effects of temper, stock thickness, grain size, and grain direction upon formability and miscellaneous design or specification properties are noted.

Minimum forming radius varies directly with stock thickness; however, no simple correlation was found relating formability with usual acceptance tests. Best combination of formability and tensile properties was obtained from material having a relatively fine grain structure. Although directionality is apparently absent in solution-treated and quarter hard strip, it may be a factor in harder tempers.   From authors' summary

743. Kenyon, J. N., The reverting of hard-drawn copper to soft condition under variable stress, *Proc. ASTM* **50, 1073–1081, 1950.**

Fatigue tests conducted on long lengths of hard-drawn copper

wire were found to develop islands of soft copper at the periphery. This reverting to large crystals was found to occur in about 30% of the heats of copper tested at room temperature and in all heats tested at 70 C. Author suggests that transmission-line failures attributed to mechanical defects in the wire may be the result of this softening phenomenon. Discussers of the paper suggested that perhaps the large grains may have been formed by the highly localized temperatures developed by repeated stressing in the plastic range.   T. J. Dolan, USA

744. Felmley, C. R., Hartbower, C. E., and Pellini, W. S., Effect of cooling rate on the aging of structural steels, *Weld. Res. Suppl.* **16,** 9, 451s–458s, Sept. 1951.

The quench and strain-aging characteristics of structural steels has been shown to be dependent on the rate of cooling from 1200 F. The steels investigated included rimmed, silicon-killed, aluminum-killed and vanadium-titanium-killed types. Rapid cooling as obtained by quenching from 1200 F enhances aging propensities regardless of steel deoxidation practice. The fully killed types develop strong aging characteristics which are otherwise absent in slow-cooled material. The development of aging effects in near-weld zones as the result of mass quenching has been demonstrated by strain-aging tests of near-weld zones for welds representative of various cooling rates.   From authors' summary

Mechanics of Forming and Cutting

745. **Krabacher, E. J., and Merchant, M. E.,** Basic factor in hot-machining of metals, *Trans. ASME,* **73,** 6, 761–769, Aug. 1951.

Hot-machining tests were conducted on Inconel X, Timkin 16-25-6 and S816 and on alloy steel AISI 3145. The 3145 steel was heat-treated to a normally unmachinable condition. Heating was done with gas flames except in one case where electric induction heating was used for heating the 3145 steel. Data are reported for room temperature, 500, 1000, and 1500 F, in general. Cutting speeds and workpiece temperatures were varied and the effect on tool forces and power consumption was measured for conditions of taking a full thickness cut from the end of a tube. Tool life tests were made using a single-tooth fly cutter tipped with sintered carbide (KM) and correlated with computed tool-chip interface temperatures. Results indicate that, in general, tool life was improved by hot machining; however, in one notable example tool life was lower at 500 F than at room temperature.

Improvements up to 80 times room-temperature tool life were observed in instances. Optimum temperatures vary depending upon the alloy.

The mechanics of hot machining is discussed. It is concluded that heating lowers the resistance of the tool to abrasion and also lowers the ability of the chip to abrade the tool. Whether improvement in tool life results depends upon which effect is the larger.   William Schroeder, USA

Hydraulics; Cavitation; Transport

746. **Spronck, R.,** Universal diagram for uniform flow in open channels (in French), *Bull. Centre Étud. Constr. génie civ. Hydl. Fluviale* 5, 359–369, 1951.

Diagram is of Nikuradse-type and based on his formulas for smooth and rough pipes, respectively. Author considers (a) very broad channels, (b) very deep, and (c) semi-circular ones, but does not deal with channels of other shapes. To illustrate use of diagram, few experiments of Bazin in a semi-circular wood channel are taken, and equivalent sand roughness is found.

To reviewer, however, it seems questionable whether just these experiments show roughness and not waviness; cf. G. H. Keulegan, "Laws of turbulent flow in open channels," *J. Res. nat. Bur. Stand.* **21,** 707–741, Dec. 1938 [RP 1151], which gives a more detailed discussion of problem in question.

                            H. Thygesen Kristensen, Sweden

747. Swain, F. E., Determination of flows in interconnected estuarine channels produced by the combined effects of tidal fluctuations and gravity flows, *Trans. Amer. geophys. Un.* **32,** 5, 653–672, Oct. 1951.

The tidal height variations are represented as a series of incremental changes in water surface height which produce small bore waves. These are assumed to travel with a celerity gh/V. The friction effects are treated by assuming the entire friction to be concentrated at a barrier (usually at the mid-length of each reach) and consisting of a head loss $K V_0^2$, K being computed from the known properties of the channel and assumed "friction factors." The gravity flows are computed by the Hardy Cross method of solving a network, and the tides are superimposed by using time increments short enough so that an integral number of them will be required for a wave to travel the length of each channel. The illustrative example given is the delta of the Sacramento and San Joaquin rivers in central California. The time interval used was 124.2 sec, or 1/720 of a mean lunar day. This made 360 intervals to the cycle, but it was found necessary to continue about 50 intervals more before the cycle began to repeat, and the first 50 intervals are not used.

The method is said to have been checked by a graphical procedure and by electric analog computers, but no estimate as to accuracy is given. It would seem to the reviewer that, due to the simplifying assumptions made, the results must be approximate only, but they are probably the best available.

                            Ralph W. Powell, USA

748. Escande, L., Two examples of transmission of water hammer waves in the closed conduit of a hydro-plant provided with a surge tank (in French), *Rev. gén. Hyd.* **6,** 60, 283–291, Nov.–Dec. 1950.

Author applies Bergeron's method to the analysis of the pressure waves induced in the closed conduit, upstream the surge tank, as a consequence of variations in the turbine regime. A surge tank without restricted orifice at the base is considered. This last device has been used sometimes in France to introduce an additional head loss in the flow into or from the surge tank.

Two cases are considered: (a) Waves produced by small but rapid rhythmic oscillations in the rate of flow in a hydro plant, and (b) waves produced by a sudden stoppage of that flow in a hydro plant with a comparatively long surge tank with respect to the penstock and a forced outlet conduit instead of customary tailwater canal. Graphical calculations and results based on various hypotheses about the rate-of-flow variations are given.

The consequences of the first case are more serious, but the second is by no means negligible. It is highly recommended to design surge tanks with a length between the penstock and the free water surface as small as possible.

Cf. also: Escande, L., "Recherches théoriques et expérimentales sur les oscillations de l'eau dans les chambres d'équilibre." Publ. Scient. Techn. Direction Ind. Aéronautiques, Paris 1943, and ibid., "Méthodes nouvelles pour le calcul des chambres d'équilibre," Dunod, 1950.

                            Armando Balloffet, Argentina

749. Kyropoulos, S., Cavitation pressures and damage, *ZAMP* **2,** 5, 406–410, Sept. 1951.

Author analyzes work of both the low-pressure and high-

pressure schools of thought on pressures associated with cavitation. He reaches the conclusion that crystal-deformation evidence for high pressures is not valid. Some reported deformation was assumed to be caused by cooling stresses in metals. Such deformation is revealed by metallographic etch characteristics of cavitation experiments, but is not caused by them. He points out that crystal boundaries are eroded during cavitation, leaving small single crystals protruding from surface. Single crystals are more easily deformed than a polycrystalline surface. Pressures for deforming single crystals are of the order of pressures for cavitation calculated by workers who favor the low-pressure hydrodynamic theory of cavitation. Author, therefore, favors low-pressure theories of cavitation.

Thomas P. Clark, USA

750. Meyer-Peter, E., Transport of sediment by flowing water (in French), *Bull. Centre Étud. Constr. génie civ. Hyd. Fluviale* 5, 279–315, 1951.

In a review of practical aspects of sediment transportation and deposition in channels and rivers, author presents résumé of his laboratory investigations (published in detail elsewhere), followed by discussions of specific applications. In first part he treats methods of transport, proposed formulas, experimental results, and similitude. In second part detailed discussion is given of (1) river structures for control of deposition and scour, (2) scour downstream of dams with either underflow or overflow, (3) effect of dams on river transport, (4) maintenance of navigable channels in rivers, and (5) design of intakes for power plants.

John S. McNown, USA

751. Durand, R., The hydraulic transportation of gravel and pebbles in pipes (in French), *Houille blanche* 6, no. B, 609–619, Oct. 1951.

Article reviews laboratory tests carried out in order to determine the flow conditions in horizontal pipes carrying various mixtures of water and gravel, for the purpose of providing information for a dredging project. According to their size, the settling-velocity law of particles can be divided in three groups. For nonspherical particles such as the gravel of the Rhône River, author shows that their form can be characterized by a volumetric coefficient [AMR 4, Rev. 3911]. Observations show the importance of saltation transport for coarse materials of the third group; at this phenomenon a characteristic riffle or dune formation is superposed. The transportation regime follows the Froude similitude. The head loss coefficient for materials of the 2nd and 3rd group increases with the concentration, but appears to be independent of the size of the particles.

Additional experimental work and fuller understanding are essential to allow extrapolation for large-sized pipes.

G. A. T. Heyndrickx, Belgium

752. Knapp, R. T., Model studies of Apra harbour, *Dock Harbour Author.* 31, 32, 366, 367, 371; 367–371, 9–14, 155–159; Apr., May, Sept. 1951.

California Institute of Technology in collaboration with U.S. Navy Bureau of Yards and Docks carried out investigations of the harbor on Guam Island in the Marianas, Pacific. A small scale open-air model was arranged at Pasadena for design of an uncompleted portion of the breakwater; horizontal scale was 1 to 960, vertical 1 to 480; the magnified vertical scale allowed more accurate wave amplitude measurements. The Pasadena basin was also used for the undistorted 1 to 480 scale model of the entrance heads. Another large scale model was necessary for investigating circulation of water within the harbor and the interchange of harbor water with sea water to avoid pollution. A

large hangar was erected at Azusa for model requiring a floor space of 120 × 120 ft. A pneumatic wave-producing machine with oscillating blowers was specially designed to cover an overall length of 100 ft. Photography was used with excellent success for fixation of pattern of the water surface configuration. This remarkably illustrated article contains short wave theory and discussion of scale distortion in models. Very valuable conclusions.

Steponas Kolupaila, USA

753. Cuénod, M., and Gardel, A., Stabilization of the oscillations of the water level in surge tanks by making the electric output temporarily dependent upon the hydraulic pressure (in French), *Bull. tech. Suisse Rom.* 76, 16, 209–218, Aug. 1950.

Two equations relating fluctuation of water in surge tank and flow variation through turbines are developed in order to study method of regulation. Principles used are Newton's laws and Bernoulli's theorem. Further study is made of effect of surge-tank cross section on oscillation of water level. Complete equation derivations are given plus numerical example. Authors conclude that stabilization of fluctuating water level in surge tanks by regulation of electric output can result in a more economical design of surge tanks.   Cameron M. Smith, USA

754. Chow, V. T., A practical procedure of flood routing, *Civ. Engng. Lond.* 46, 542, 586–588, Aug. 1951.

Article outlines simple graphical method of routing flood waves through river reaches or reservoirs. It is assumed that equation of continuity describes flow and that dynamic effects are negligible. Method then applies only to cases where flow changes gradually with time, and storage in reach is function of discharge only. Curves of storage and inflow to a reach are set up from behavior of a known flood wave. Hypothetical flood can then be solved by graphical method using these curves. Results of floods greater than record and changes in river configuration can only be approximated.

W. Douglas Baines, Canada

755. Weale, R. A., Surface tension of liquid metals, *Nature* 168, 4269, p. 343, Aug. 1951.

756. Hacker, P. T., Experimental values of the surface tension of supercooled water, *NACA TN* 2510, 20 pp., Oct. 1951.

A total of 702 individual measurements of surface tension of triple-distilled water were made in the temperature range 27 to −22.2 C with 404 of these measurements at temperatures below 0 C. The increase in magnitude of surface tension with decreasing temperature, as indicated by measurements above 0 C continues to −22.2 C. The inflection point in the surface-tension-temperature relation in the vicinity of 0 C, as indicated by the International Critical Table values for temperatures down to −8 C, is substantiated by the measurements in the temperature range, 0 to −22.2 C. The surface tension increases at approximately a linear rate from a value of 76.96 ± 0.06 dynes per cm at −8 C to 79.67 ± 0.06 dynes per cm at −22.2 C.

From author's summary

757. Elton, G. A. H., Contact angles and surface tensions in liquid-solid systems, *J. chem. Phys.* 19, 8, p.1066, Aug. 1951.

Author assumes Antonoff's rule (the interfacial tension between two mutually saturated liquids is equal to the difference between the individual surface tensions of the saturated liquids) to hold also for solid-liquid interfaces in mutually saturated phases. Thus he can eliminate the interfacial tension in the expression for the contact angle of a liquid against a solid. By measuring this latter angle and knowing the surface tension of

the liquid, one can calculate the surface tension of the solid. A few experimental results corroborate this hypothesis and procedure, which show that consistent results are obtained for the surface tension of solid paraffin when the procedure outlined is carried out for three different hydroxylic liquids.

Bruno W. Augenstein, USA

758. Iwasaki, H., Measurement of viscosities of gases at high pressure. I. Viscosity of air at 50°, 100° and 150° C up to 200 atmospheres, *Sci. Rep. Res. Inst. Tôhoku Univ. (A)* **3**, 2, 247-257, Apr. 1951.

The viscosity of air is consistently determined over a wider range of pressure plus temperature than heretofore. A Macwood-type oscillating disk viscometer with large axial clearance is used. Results are consistent and believed accurate to 1%, though deviate 3 to 4% from Golubev's values. They will be compared with Enskog's theory subsequently. Typical smoothed values are, in micropoises:

Atm abs	1	20	50	100	150	190	200
50 C	197.3	199.6	204.3	214.2	227.6	240.0	243.7
100 C	219.5	221.0	223.6	230.9	241.0	250.7	253.7
150 C	239.0	240.0	241.2	245.6	253.8	261.9	264.0

The values at 1 and 200 atm are extrapolated.

C. F. Bonilla, USA

759. Fortier, M., Some new results on singular losses of head (in French), *Houille blanche* **6**, no. B, 598-601, Oct. 1951.

With instruments now in use, it is possible to measure the total mean pressure in a pipe section only when the mean velocities are approximately parallel to a determined direction. Author defines total pressure as $P = 1/\overline{Q} \int\int_\sigma (p + \rho g h + \rho V^2/2) V_n d\sigma$, in which Q is the mean rate of flow; p the pressure; ρ specific mass; h the elevation of the elementary area $d\sigma$ above a fixed reference level; V the velocity, and V_n the velocity, normal to the element $d\sigma$. The bars denote temporal mean values.

If a singularity disturbs the downstream flow, the loss of head can be determined only through averaging of total pressure in a section downstream the perturbed flow. The head loss is then the sum of that due to the singularity and that in the perturbed downstream sections.

Author shows how it is possible to separate these two elements and to determine the head loss of the singularity itself. In this way author adds to the head-loss coefficient K, two other formulas giving the variations of the Coriolis coefficient α (kinetic energy coefficient) and the momentum coefficient β in the singularity outlet section.

Some examples of application are given. Discussion by Danel, Gaden, and Fortier closes the paper.

From author's summary by Armando Balloffet, Argentina

Incompressible Flow: Laminar; Viscous

(See also Revs. 621, 747, 829, 831, 843, 873, 925, 832)

760. Vladimirsky, S., Nonuniform plane motion of an infinitely thin plate (in French), *C. R. Acad. Sci. Paris* **231**, 1, 30-32, July 1950.

This paper considers the fluid motion produced by a flat plate moving with given time-dependent translation and rotation, while taking into account a vortex sheet formed at the trailing edge. The basic assumption, which the author considers appropriate for motions of small duration, is that the vortex sheet of unknown strength is of the same shape as the (known) curve described by the trailing edge of the plate, and that elsewhere the

fluid motion is irrotational. Author presents without proof an integral equation for the vortex strength, which is derived by adding to the above assumption the condition of finite velocity at the edges of the plate and of zero total circulation about plate and vortex sheet. Expressions for the potential and pressure are given in terms of the unknown vortex distribution.

Courtesy of Mathematical Reviews　　　　D. Gilbarg, USA

761. Cheers, F., Raymer, W. G., and Douglas, O., Tests on a "Lighthill" nose-suction aerofoil in the N.P.L. 4-ft. No. 2 wind tunnel, *Aero. Res. Counc. Lond. Rep. Mem.* 2355, 7 pp., Apr. 1947, published 1951.

A series of tests on an 8.6% thick nose-suction airfoil designed by Lighthill has been made in the 4-ft No. 2 wind tunnel at the National Physical Laboratory, at Reynolds numbers of 0.385 and 0.577×10^6. The results show that the wing stalls at $\alpha \sim 13$ deg ($C_L = 1.12$) without suction, the lift coefficient at the stall increasing approximately linearly with suction quantity and reaching 1.93 at $C_Q = 0.019$ and 23-deg incidence.

From authors' summary

762. Berndt, S. B., An approximate method for determining the incompressible, irrotational flow around a symmetrical wing section, *Roy. Inst. Technol. Div. Aero. Stockholm KTH-Aero TN* 7, 18 pp., 1949.

Starting from the Theodorsen method of mapping conformally an arbitrary wing section upon a circle, an approximate expression is obtained for the singularity distribution on the axis of a symmetrical wing section in incompressible, irrotational flow. A method is suggested for computing the stream function and the velocity field around the wing section by expanding the singularity distribution into a series, and the functions necessary are tabulated.

From the author's summary by E. B. Klunker, USA

763. Lord, W. T., The incompressible potential flow past two-dimensional aerofoils with arbitrary surface suction, *Aero. Res. Counc. Lond. curr. Pap.* 56, 35 pp., 9 figs., June 1950, published 1951.

Problem is solved by application of conformal transformation to the solution derived for the corresponding problem for the unit circle. Several simple examples involving a Joukowsky airfoil are given. It is found that the effect on the main stream potential flow of distributed suction for the purpose of boundary-layer control is negligible.　　　　John R. Spreiter, USA

764. Landweber, L., The axially symmetric potential flow about elongated bodies of revolution, *David W. Taylor Mod. Basin Rep.* 761, 61 pp., Aug. 1951.

An iteration formula for Fredholm integral equations of the first kind [see AMR 5, Rev. 16] is applied in two new methods for obtaining the steady, irrotational, axisymmetric flow of an inviscid, incompressible fluid about a body of revolution. In the first method a continuous, axial distribution of doublets is sought as a solution of an integral equation of the first kind. A method of determining the end points and the initial trends of the distribution, and a first approximation to a solution of the integral equation are given. This approximation is then used to obtain a sequence of successive approximations whose successive differences furnish a geometric measure of the accuracy of an approximation. When a doublet distribution has been assumed, the velocity and pressure can be computed by means of formulas which are also given.

In the second method the velocity is given directly as the solu-

tion of the integral equation of the first kind. Here also a first approximation is derived and applied to obtain a sequence of successive approximations. In contrast with the first method, which, in general, can give only an approximate solution, the integral equation of the second method has an exact solution.

Both methods are illustrated in detail by an example. Results are compared with those obtained by other known methods.

From author's summary by R. C. Roberts, USA

765. Vladimirsky, S., Theory of unsteady motion of a flat plate by the potential method (in French), *C. R. Acad. Sci. Paris* 233, 5, 352–354, July 1951.

Author studies the flow around a flat plate which starts from rest with forward velocity U, while being displaced vertically with variable velocity $V(t)$. A velocity field is produced in terms of simple algebraic integrals, and the character of these integrals on the corresponding Riemann surfaces is discussed. The vorticity in the wake satisfies Wagner's integral equation.

The idea of the paper is interesting, but it appears to reviewer that author has neglected the condition that the pressure must be continuous across the wake. In consequence, his results cannot, in general, be expected to be in agreement with accepted theory.

A. Robinson, Canada

766. Timman, R., The aerodynamic forces on an oscillating aerofoil between two parallel walls, *Appl. sci. Res.* A3, 1, 31–57, 1951.

An exact solution is given for the aerodynamic forces on an oscillating airfoil between two parallel walls. The solution is made by a method analogous to Theodorsen's method of the velocity potential and conformal transformation in the case of the free airfoil. Notation similar to the Küssner notation is used in the development, and it is shown that the results approach the Küssner coefficients when the tunnel walls are assumed to be at an infinite distance from the airfoil.   John E. Stevens, USA

767. Ludwieg, H., The fully developed channel flow in a rotating system (in German), *Ing.-Arch.* 19, 4/5, 296–308, 1951.

The flow in a square cross-section channel rotating about an axis at right angles to the channel axis is investigated. It is shown that, at sufficiently high rotational velocities, the flow field can be separated into two domains, namely, a boundary-layer flow on those side walls perpendicular to the axis of rotation, and a core flow filling the remaining portion of the channel. It is found that, for laminar flow, the friction drag in a channel in a rotating system differs considerably from that in a stationary channel. To check the calculations, pressure-drop measurements were made in a helical channel rotating about its axis. The measured friction coefficients were in agreement with those predicted even when the core flow was turbulent. This is attributed to the existence of laminar boundary-layer flow in the rotating system at much higher Reynolds numbers than would be possible in a nonrotating channel flow.   Gerald E. Nitzberg, USA

768. Okabe, J., Approximate calculations of laminar jets, *Rep. Res. Inst. Fluid Engng. Kyushu Univ.* 5, 1, 1–22, Sept. 1948.

Using equations derived by Yamada and Okabe [title source, 4, 1, 1947] author analyzes flow in laminar submerged jets. For cases of uniform velocity distribution issuing from two-dimensional slot and circular orifice, equations are solved and stated in integral form. Velocity profiles are computed for representative sections and compared to the approximate solutions of other workers. Resulting curves show that this method gives a closer approximation than method using von Kármán momentum theorem, especially near the mouth of the jet where no discon-

tinuity is present in author's solution. In second part of paper, author introduces a less laborious method of solution of above-noted equations. Resulting curves for velocity profiles are a close approximation to the more exact solution.

W. D. Baines, Canada

769. Panasenkov, N. S., Effect of the turbulence of a liquid jet on its atomization (in Russian), *Zh. tekh. Fiz.* 21, 2, 160–166, Feb. 1951.

Photographic measurements were made of the length before atomization of a water jet discharging into the atmosphere, as a function of Reynolds number based on orifice diameter. The length increased up to $R = 4200$ and then dropped off sharply, presumably because of the establishment of turbulent flow. Average drop size after atomization of a turbulent jet was found to be almost independent of Reynolds number, but approximately proportional to orifice diameter. The data were fitted by the empirical curve $D_{drop} = 6D_{orifice}/R_e^{0.15}$.

Leon Trilling, USA

770. Barberon, P., Introduction to the study of wings in transition flow (in French), *Bull. Assn. tech. marit. aéro.* no. 49, 167–185, 1950.

First part of paper is devoted to obtaining beginning and variation of circulation for a two-dimensional lifting wing in incompressible flow, started from rest and accelerated rectilinearly. It is assumed that the wing leaves in its wake a surface of discontinuity whose local vortex strength is determined by the rate of change of circulation along its flight path. The formation of such a wake and the distribution of vortexes in the wake are determined by the assumption that the flow at each instant conforms with the Kutta condition. Method used here for computing circulation by two-dimensional potential theory is much the same as Wagner's treatment of this problem [*ZAMM* 5, 1, 17–35, Feb. 1925; see also Durand, "Aerodynamic theory," vol. II, pp. 289–293].

Second part of paper is devoted to obtaining expressions for the forces and moments acting on wing due to this variation in circulation.   S. Lampert, USA

771. Wiegel, R. L., and Johnson, J. W., Elements of wave theory, *Proc. First Conf. Coast. Engng.,* 5–21, 1951.

Authors summarize theories of water waves advanced by Gerstner (1802), Airy (1845), Stokes (1847 and 1880). Engineers interested in water waves will find the graphs more helpful than the text. The graphs give relations between wave length, velocity, period, depth. Data are also presented on the orbits of water particles and theories compared with experiments [Morrison, *Trans. Amer. geophys. Un.* 32, p. 201, 1951]. The third approximation of Stokes theory gives the best fit so far as the scatter of the experimental results allows a decision.

J. M. Jackson, Scotland

772. Mason, M. A., The transformation of waves in shallow water, *Proc. First Conf. Coast. Engng.,* 22–32, 1951.

Author discusses variation of wave velocity, height, and wave length as a train of harmonic waves approaches a slowly shelving shore. Work is largely a summary of reports of the Beach Erosion Board and of the Scripps Institute of Oceanography, together with recent laboratory results [AMR 4, Rev. 1853]. Equations (3) and (4) appear to be based on Gaillard's modified trochoidal theory. This theory is inexact but can be used as a rough approximation. The authors have stated that the orbits of water particles are not closed and are not ellipses. The question of the localization of energy is difficult. Reviewer prefers to regard

the energy as traveling with the group velocity rather than to consider a fraction traveling with the wave velocity and the remainder localized. Agreement between theory and experiment is fairly good considering the difficult nature of the experiments, observations, and the number of extraneous factors involved.

J. M. Jackson, Scotland

773. Bory, Ch., Velocity distribution in the neighborhood of a wall moving tangentially to the fluid flow (in French), *C. R. Acad. Sci. Paris* **233**, 5, 350–352, July 1951.

Author obtains, in numerical form, the boundary-layer velocity distribution for a semi-infinite flat plate lying parallel to a uniform stream; the flat plate being fixed for $z < l$ and moving parallel to itself with a speed equal to that of the free stream for $x > l$.   Peter Chiarulli, USA

774. Kinoshita, M., Wave resistance of a sphere in shallow sea, *J. Zosen Kiokai* **73**, 19–38, July 1951.

Paper deals with the wave resistance experienced by a small sphere in stationary motion below the free surface of a fluid of finite depth. Author claims to have obtained a new formula for the wave resistance. Reviewer notes that this formula agrees with Havelock's corrected one [*Proc. roy. Soc. Lond. (A)* **144**, 514–521, Jan. 1934].

Results of some numerical calculations are given in the form of curves.   J. K. Lunde, Norway

775. Sacks, A. H., and Spreiter, J. R., Theoretical investigation of submerged inlets at low speeds, *NACA TN* 2323, 48 pp., Aug. 1951.

Analysis is made of incompressible flow field in a submerged air inlet (say, on fuselage of jet-propelled airplane) for aircraft internal flow systems. Pressure-recovery and drag characteristic estimates are presented to determine the applicability of this type inlet to aircraft.

Submerged inlet does not operate in the free-stream air, and the air passing through it suffers pressure losses due to vortex formation and the boundary layer. Magnitude of these losses depends on design of the approach ramp.

A simplified theoretical approach is taken to determine the behavior of the boundary layer, the vortex flow, and their interactions. The effects of mass flow and compressibility are discussed. The above are complemented by wind-tunnel tests and visual flow studies.

Authors conclude that an optimum mass-flow ratio and divergence angle for a straight ramp exists which would give minimum total-pressure loss. Report should prove useful to aircraft designers.   Irvine I. Glass, Canada

776. Nagamatsu, H. T., Circular cylinder and flat plate airfoil in a flow field with parabolic velocity distribution, *J. Math. Phys.* 30, 3, 131–139, Oct. 1951.

An analysis is presented of a two-dimensional incompressible, nonviscous flow over a circular cylinder with and without circulation in a free-stream parabolic velocity distribution of the form $u = U_0(1 + k(y^2/c^2))$, $v = 0$ where k is a nondimensional constant, c a representative length of the body, and U_0 the undisturbed flow velocity along the x-axis. Effects of free-stream parabolic and linear velocity distribution over a flat plate at angles of attack are also considered. A solution of the form $\psi = \psi_0 + \psi_1$, where ψ_0 is the free-stream function, ψ_1 disturbance stream function due to the body, is utilized. The velocity-curvature factor k, the disturbance due to the body, and the deflection of the streamline are assumed small. In the case of the parabolic velocity distribution the linear term of a Taylor

series expansion (from the free-stream distribution) of the vorticity distribution across the streamline is used. Thus it is found that ψ_1 must satisfy Laplace equation and the appropriate boundary conditions. Conclusions are that, in the case of a circular cylinder and parabolic free-stream velocity, the effect of vorticity on the velocity distribution along the negative x-axis (cylinder located at origin) is to move a given decrease in free-stream velocity farther away from the origin than for the case of uniform flow. In the case of a circular cylinder with circulation, the pressure coefficient is made more negative than for flow without vorticity (for the same circulation). The flat plate with parabolic velocity distribution has a greater maximum velocity on the upper surface at a particular angle of attack than the case without vorticity at the same angle of attack and velocity U_0. In the case of the flat plate in a linear free-stream velocity distribution, the maximum velocity on the upper surface for a given angle of attack is decreased from that without vorticity. The latter results of the flat plate show tendencies of opposite sign in comparison with the previous case of parabolic velocity distribution. This result appears to need further investigation since the flat plate with a linear free-stream velocity distribution can be calculated exactly without utilizing the assumptions used in all the other cases.   Henry G. Lew, USA

777. Hatanaka, H., On the general solutions of the two-dimensional Oseen's equations of a viscous fluid (in Japanese), *Rep. Inst. Sci. Technol. Tokyo,* **5**, 1/2, 45–48, Aug. 1951.

General solutions of the Oseen equation for the two-dimensional viscous flow are written, with usual notation, as

$$u = \partial\varphi/\partial x + e^{2kx}\partial\chi/\partial x = \partial\psi/\partial y$$
$$v = \partial\varphi/\partial y + e^{2kx}\partial\chi/\partial y = -\partial\psi/\partial x$$

where $k = U/2v$, $\psi = \psi_0 + (2k)^{-1}e^{kx}\psi^*$, and $\Delta\varphi = \Delta\psi_0 = 0$ $\chi^* = 2ke^{kx}\chi$ and ψ^* satisfy the same equation $(\Delta - k^2)(\chi^*, \psi^*) = 0$ and correspond, in the limiting case $k \to 0$, to the velocity potential and stream function of nonviscous flow. For flow past an arbitrary cylinder, χ^* and ψ^* are obtained in polar coordinates (r, θ) as

$$\chi^* = c_1\Theta_1 + \Sigma K_n(\xi)(a_n \cos n\theta + b_n \sin n\theta)$$
$$\psi^* = c_2\Theta_2 + \Sigma K_n(\xi)(A_n \cos n\theta + B_n \sin n\theta)$$

with

$$\Theta_{1,2} = e^{\pm\xi\cos\theta}\{\theta + 2\Sigma(\pm 1)^n n^{-1}\xi[K_1(\xi)I_n(\xi) + K_0(\xi)I_n{}'(\xi)] \sin n\theta\},$$

where $\xi = kr$, and K_n, I_n are Bessel functions and a_n, b_n, A_n, B_n, c_1, c_2 are arbitrary constants. Hence, asymptotic expressions are derived for the velocity and vorticity at infinity.

Isao Imai, Japan

Compressible Flow, Gas Dynamics
(See also Revs. 826, 828, 832, 833, 844, 865, 871, 878, 881, 899, 901)

778. Dorrance, W. H., Nonsteady supersonic flow about pointed bodies of revolution, *J. aero. Sci.* 18, 8, 505–511, 542, Aug. 1951.

Velocity potential for case of slender bodies oscillating periodically in a supersonic uniform stream is presented. Stability derivatives for low frequency oscillations are determined by expansion in powers of the reduced frequency and retaining only first-order terms. Four cases of steady and nonsteady supersonic motion are treated: (1) Harmonic pitching about a point x_0; (2) harmonic normal oscillations in pitch; (3) steady pitch-

ing about a point x_0; and (4) steady angle of attack. Equations for the slender-body theory normal force and pitching-moment stability derivatives are determined. Two types of bodies of revolution are examined to demonstrate how the aerodynamic stability derivatives may be varied by changing the contour of the body.

From author's summary by A. van Heemert, Holland

779. Tucker, W. A., and Nelson, R. L., Theoretical characteristics in supersonic flow of two types of control surfaces on triangular wings, *NACA Rep.* 939, 16 pp., 1949.

Linearized theory is used to find the lift, rolling moment, pitching moment, and hinge-moment characteristics of two types of control surfaces on thin triangle wings. Types considered are constant-chord partial-span flaps, either outboard or inboard, and full triangular-tip flaps with planform geometrically similar to the wing (the latter for case of supersonic leading edge only). Conclusions are that triangular-tip flaps are better than constant-chord type for this application because of their generally higher effectiveness and less serious hinge-moment problems.

J. S. Isenberg, USA

780. Melkus, H., On the detached shock wave (in German), *Ing.-Arch.* 19, 3, 208-227, 1951.

Author derives formulas for the velocity distribution after a detached shock wave and suggests that the velocity field between the shock and the body can be computed by a series expansion. As an application, author computes the distance between the shock and the body along the axis of symmetry. However, only terms up to second order are included in the expansion and no investigation about the convergence is made.

Paper includes many useful formulas; however, in a few of them the detached shock has been approximated with a hyperbola, using the Mach lines as asymptotes. As is pointed out by several other authors, the detached shock does not asymptotically approach the Mach lines, but rather the shock angle has an asymptotic value equal to the Mach angle.

G. Drougge, Sweden

781. Cabannes, H., Theoretical determination of the fluid flow behind a detached shock wave (in French), *ONERA Note tech.* 5, 27 pp., 1951.

Author first inverts the usual procedure by assuming quartic shock family, $x = r^2/2R + \lambda r^4/4R^3$ (x, r cylindrical coordinates; with origin at shock apex; R radius of shock curvature at apex) and studying corresponding stream surfaces as a function of the parameter λ. This is accomplished by "brute force," expanding velocity components, pressure, density, and stream function ψ, in double series up to fourth powers around the shock apex and using shock and flow equations (differentiating the latter as well) at $x = 0$ to determine the constant coefficients at free-stream Mach number of 2, 3, 4, 5, 6, 10, ∞. Identifying $\psi(x, r) = 0$ with the assumed obstacle shape $x = h + r^2/2k + \tau r^4/4k^3$, approximations of fourth order for h, the distance to body apex, of second order for K, the radius of curvature of body at apex, and of first order for the parameter τ, are obtained. The procedure leads to well-behaving obstacle surfaces for values of $\lambda < \lambda_0(M)$; for $\lambda > \lambda_0(M)$, tracings of $\psi = $ constant yield hollow (choked) bodies.

Brief consideration is given to solving the same system inversely for λ, h, and R, in terms of known body τ and K. For a range of τ's, three solutions are possible, i.e., three shock curves can occur for the same body. In spite of admittedly poor approximation in the equation for τ, author attaches significance to the triple possibility and likens it to the indeterminacy between strong and weak shocks when attached to sharp obstacles.

The branch of solutions retained corresponds to minimum entropy rise. However, for this branch the shock shape characterized by λ is practically insensitive to variation of obstacle shape (τ). In reviewer's opinion, such behavior in approximate solutions is usually indicative of inaccuracy and should suggest caution in interpreting results obtained by assuming shock shapes.

M. V. Morkovin, USA

782. Herpin, A., General theory of shock waves (in French), *Mém. Artill. fr.* 24, 4, 851-897, 1950.

Author presents a complete survey of shock waves and their application in a rather short paper. In spite of this, paper is so concise that those who are less familiar with the subject matter, as well as those who are better acquainted with shock waves, will profit from it. It is prepared in three chapters. Chapter I concerns shock waves of finite amplitude (small disturbances). General equation of wave propagation, method of characteristics, motion of a gas in a tube, and related problems are treated. In chapter II, Hugoniot's theory, fundamental equations of shock waves are reviewed; such problems as speed of shock waves, instability of shock waves are treated. Chapter III gives kinetic theory of shock waves. Also in this chapter the physical significance of the principal problems treated in chapters I and II are stressed. Paper is more than a mere survey of the field; it is intended as a tool for engineers and physicists alike who encounter various problems involving shock waves.

Ahmed D. Kafadar, USA

783. Stocker, P. M., On a problem of interaction of plane waves of finite amplitude involving retardation of shock-formation by an expansion wave, *Quart. J. Mech. appl. Math.* 4, part 2, 170-181, June 1951.

The comparison between the ease with which shock waves are produced experimentally in a flowing gas and the difficulty of predicting and localizing such a wave mathematically is a permanent lesson of modesty for the pure mathematicians. Considering the simple case of the interaction of a compressive receding wave and an expanding advancing wave, author points out the delay of formation of the shock wave due to the rarefaction. An interesting formula is given for the computation of the time at which shock formation occurs. F. H. van den Dungen, Belgium

784. Mott-Smith, H. M., The solution of the Boltzmann equation for a shock wave, *Phys. Rev.* (2) 82, 6, 885-892, June 1951.

Previous calculations of the internal constitution of a shock wave are discussed. Calculations by the method of Chapman and Enskog in which the velocity distribution is considered to be a perturbation of the Maxwell distribution have culminated in the work of Wang-Chang, who has calculated terms up to third order in the velocity-distribution function. She has found that this method of approximating to the real distribution function either diverges or converges very slowly for strong shock waves, although it is apparently adequate for weak shock waves. Author proposes for strong shock waves to adopt what he calls a bimodal distribution function, i.e., a superposition of the equilibrium-distribution functions appropriate to the regions ahead of and behind the shock wave. The relative concentrations of the two component distributions is assumed to vary through the shock waves. The transport equation is then simplified to an ordinary differential equation by this assumption and solutions are given for shocks of various strengths. To check the adequacy of the assumed bimodal distribution function, author calculates the relative concentrations of the sub- and supersonic components using both equations for u^2 (u is the molecular velocity normal to the shock) and u^3. He finds that the agreement between these is quite

good. He also checks the validity of this distribution function by examining the rate-of-change of the fraction of the supersonic component of the distribution with time at a stationary point in the solution obtained. This quantity should be zero if his solution is exact, since it is intended to represent a steady shock wave. In view of the fact that the time rate-of-change is in reality quite slow, it seems that this type of distribution function is close to the truth. The agreement of Mott-Smith's theoretical results with the experimental results of Green, Cowan and Hornig [*J. Chem. Phys.* 19, 4, p. 427, 1951; see Figs. 4 and 5] is better than that of any of the previous theories.   Arthur Kantrowitz, USA

785. Stewartson, K., On the interaction between shock waves and boundary layers, *Proc. Camb. phil. Soc.* **47,** part 3, 545–553, July 1951.

The interaction between a weak normal shock wave and the laminar boundary layer on a flat plate is investigated. It is shown that if the shock wave is of strength ϵ (defined as half of the velocity difference before and after shock, expressed in terms of the mean velocity before and after shock) then separation occurs when ϵ is of the order of Reynolds number to the $-2/5$ power, wherein the Reynolds number is based on a length which for very weak shocks is nearly equal to the length of run from the leading edge of the plate to the position of normal shock.

H. Julian Allen, USA

786. Oppenheim, A. K., A contribution to the theory of the development and stability of detonation in gases, Ann. Meeting ASME, Atlantic City, 1951, Paper no. 51—A-23, 9 pp.

Author's problem was treated in a more complete way by Busemann ["Problems of detonation," *Mitt. dtsch. Akad. Luft-fahrforsch.*, 1941], giving physical explanation of entropy minimum by normal detonation and distinguishing the following theoretical cases: voluntary and forced detonation, prepared and spontaneous combustion. Author agrees in dealing with prepared combustion and in explaining the transition from combustion to detonation by a nonstationary system of shock wave and combustion front.   F. Schultz-Grünow, Germany

787. Imai, I., An approximate method of calculating compressible fluid flow past a thin aerofoil, *J. phys. Soc. Japan* **3,** 346–351, 1948.

788. Imai, I., On a new method of approximation for treating compressible fluid flow, *J. phys. Soc. Japan* **3,** 352–356, 1948.

These two papers present two closely related approximate methods for computing nearly parallel subsonic gas flows past airfoils. Let $u = q \cos \theta$, $v = q \sin \theta$ be the velocity components, c the local speed of sound, and $\mu^2 = 1 - q^2/c^2$. First method: Assume that v is small compared to u, so that $(q/c)^2$ may be replaced by $(u/c)^2$, and c may be treated as a function of u. Neglecting small terms and setting $\omega = \int \mu du$, (1) $\xi = x$, $\eta = \int \mu dy$, it turns out that $\omega + iv$ is an analytic function of $\xi + i\eta$. In order that (1) have a meaning, an assumption for μ inside the profile must be made. The flow problem is now reduced to a boundary-value problem for analytic functions. An iteration scheme involving the familiar cot- integral is used for solving this problem. (No convergence proof is given.) Second method: Using the assumption that θ is small and setting $\tau = \int \mu d \log q$, it is shown that $\theta + i\tau$ is (approximately) an analytic function of $\xi + i\eta$. If one sets $\mu \equiv 1$ inside the profile, it turns out that a solution of an incompressible-flow problem yields at once a solution for the approximate problem considered. For points on the profile, a

"universal" velocity-correction table can be given. Both methods are illustrated by numerical examples. For supersonic flows, the same methods lead to the wave equation. [As in all thin-wing theories, the basic assumptions break down near a stagnation point, but the author does not discuss this difficulty.]

L. Bers, USA

789. Callaghan, E. E., and Ruggeri, R. S., A general correlation of temperature profiles downstream of a heated-air jet directed perpendicularly to an air stream, *NACA TN* 2466, 37 pp., Sept. 1951.

An empirical method is presented to determine a dimensionless "penetration coefficient" for title problem. An analysis based upon previous investigations of the authors is given. By means of tests in a "two-dimensional" tunnel of 2 in. $\times$ 20 in., algebraic relations between characteristic parameters are determined. (Free-stream velocities ranged from 160 to 360 fps. The jet entered the airstream through thin-plate orifices with diameters of 0.25 in. to 0.625 in. at ratios of jet total pressure to outlet static pressure from 1.2 to 3.7, and at total temperatures from 200 F to 400 F.) Using the so found penetration coefficient, temperature profiles are computed and compared with experimental data. Agreement is satisfactory. Deviations decrease with distance from orifice. Authors are aware of restrictions in using "two-dimensional" tunnels. They briefly outline their opinion about the steps to be taken for the extrapolation of results to three-dimensional problems.   Heinrich J. Ramm, USA

790. Oswatitsch, K., The compressibility effect on slender bodies of revolution in subsonic and supersonic flow (in German), *Arch. Math.* **2,** 6, 401–404, 1949/1950.

791. Ludford, G. S. S., The behavior at infinity of the potential function of a two-dimensional subsonic compressible flow, *J. Math. Phys.* **30,** 3, 117–130, Oct. 1951.

Author extends the hodograph method developed by Bergmann [*Trans. Amer. math. Soc.* **62,** 3, 1947] and finds the expansion at infinity of the potential in two-dimensional subsonic compressible flow. This is used to obtain expressions for lift, drag, and pitching moment on a subsonic airfoil.

Maurice Holt, England

792. Katzoff, S., and Hannah, M. E., Further comparisons of theoretical and experimental lift and pressure distributions on airfoils in cascade at low-subsonic speed, *NACA TN* 2391, 24 pp., Aug. 1951.

Report compares calculated lift and velocity distributions for five compressor-type cascades of highly cambered NACA 6-series airfoils. Experimental data are obtained by improved technique described in *NACA TN* 2028. Comparisons between theory and experiment are much better than those previously published in *NACA TN* 1376. The experimental lift coefficients were less than the theoretical value by amounts varying from 0.2 to 0.4 for lift coefficient of the order of 0.9, the larger differences occurring for the more highly cambered airfoils and conditions of large pressure rise. Pressure distribution calculated by equating the circulation to the experimental value and neglecting the Kutta condition agreed well with the experimental distribution, provided the pressure rise was not so high as to cause separation of the airfoil boundary layer.   Robert M. Crane, USA

793. Haack, W., A method of characteristics for the approximate calculation of unsymmetric supersonic flow around ring-shaped bodies (in German), *ZAMP* **2,** 5, 357–375, Sept. 1951.

A method of characteristics has been developed to calculate the linearized supersonic flow over inclined bodies of revolution. A similar method had been developed earlier by Sauer and Heinz (unpublished). The reviewed method seems to be faster for calculating the unsymmetrical part of the flow than the method of Sauer-Heinz. Author points out that the usual way to calculate such flows is with the singularity method, which is very cumbersome for more complicated bodies, such as rings around central bodies. Such flows can easily be calculated with the linearized characteristic method. As an example, the flow over some simple combinations of rings and central bodies has been calculated. Tore Gullstrand, Sweden

794. Braun, W. H., and Klein, M. M., Calculation of higher approximations for two-dimensional compressible flow by a simplified iteration process, *NACA TN* 2511, 54 pp., Oct. 1951.

Convergence of iterative-type series solutions for velocity potential of flow past thin airfoils at high subsonic speeds is investigated by calculating the first six terms of such series for ellipse and first four terms for Kaplan bump. Labor of calculation is kept reasonable by using small perturbation transonic limiting form of differential equation for potential and expanding in terms of transonic similarity parameter $K = \tau M_0^2 (1 + \frac{1}{2} (\gamma - 1) M_0^2)/(1 - M_0^2)^{3/2}$ [see Perl, W., and Klein, M. M., AMR **4**, Rev. 2605]. Additional simplification of d.e. is made by restricting probable validity of results to region near midchord. Main results, for perturbation velocity φ_x, are

$$\beta \varphi_x/\tau = \Sigma_{n=0}^{m} a_n K^n \qquad [*]$$

where $\beta = (1 - M_0^2)^{1/2}$. The a_n are given as functions of position in flow field and $m = 5$ and 3 for ellipse and Kaplan bump, respectively. Graphical inspection of a_n at mid-chord on contour suggests their ultimate increase with n as a geometric series and hence limiting values K_l for series convergence of 0.56 for ellipse and 0.387 for Kaplan bump, which values yield local supersonic regions.

Reviewer's remarks: The graphically suggested geometric series-type increase of a_n would appear to imply that expansion [*] could be completed by adding the term $a_m K^{m+1}/(K_l - K) \ldots$ [**]. Such a term, however, besides giving very sizable corrections for $K < K_l$, makes $\varphi_x \to \infty$ as $K \to K_l$. A possibly more realistic estimate of remainder term is obtainable from assumption that velocity is finite but its derivative by K infinite at $K = K_l$ [see Perl, W., AMR **4**, Rev. 2589]. Then, for example, [*] could be differentiated by K twice, geometric-type remainder term [**] added, result integrated to give logarithmically infinite velocity derivative at $K = K_l$ and integrated again to give finite velocity at $K = K_l$. Remainder term thus obtained is

$$m(m - 1) \,_0\!\int^K \,_0\!\int^{K_1} [K_2^{m-1}/(K_1 - K_2)] dK_2 dK_1.$$

Evaluation of K_l from [*] need not be altered if the a_n really do vary geometrically ultimately, since same K_l would then be obtained from second derivative series. Simplification of differential equation for potential is now such that analytical estimate of nth term of [*] to settle convergence question definitely would not appear impracticable. William Perl, USA

795. Legendre, R., Laminar airfoil for subsonic velocities (in French), *Rech. aéro.* no. 22, 3-6, July-Aug. 1951.

A theory is developed for designing airfoils having an approximately prescribed velocity distribution in subsonic flow. The method, which was previously applied to turbomachine blades, involves the assumption of a linear pressure-density relation. It is particularly suited to the calculation of laminar-flow airfoils. The treatment is brief; numerical applications are to be published later. Milton D. Van Dyke, USA

796. Mesniére, M., Laminar profile at subsonic speed (in French), *Rech. aéro.* no. 23, 23-26, Sept.-Oct. 1951.

Paper is a first numerical application of an analytical design method proposed by Legendre [see preceding review]. Method itself is an extension to airfoils of a former one [AMR **3**, Rev. 1536] intended for blade lattices. Since present paper accounts only for numerical calculation procedure, it is necessary to know of original papers. A profile of 8% thickness with prescribed velocity distribution is calculated for $M = 0.8$ and $C_L = 0.407$, stations of velocity maxima being located at 70% chord, thus suggesting good laminar properties. Author states that design method is convenient for determining a great variety of profiles suitable for high subsonic velocities. Coordinates of calculated profile are given in a table. Pierre Schwaar, Switzerland

797. Miles, J. W., Transient loading of wide delta airfoils at supersonic speeds, *J. aero. sci.* **18**, 8, 543-554, Aug. 1951. See AMR **4**, Rev. 792.

798. Mark, R. M., Application of the extended Kármán-Tsien method for the generation of conventional airfoils in two-dimensional subsonic compressible flow, *Bull. Univ. Wash. Engng. Exp. Sta. Seattle* no. 118, Aero. Ser. no. 1, 7-11, 1951.

Paper discusses the inverse problem of compressible flow in relation to the generation of conventional airfoils by mapping functions. The von Kármán-Tsien method, as extended by Lin for circulatory flow, is applied to the particular case of a conventional airfoil generated from a circle. Numerical calculations are carried through for a chosen set of constants when Lin's conditions are satisfied. Robert Simon, USA

799. Guderley, G., Axial-symmetric flow patterns at a free-stream Mach number close to one, *AF tech. Rep.* 6285, 26 pp., Oct. 1950.

Author employs variable combination similar to boundary-layer type to get particular solution of nonlinear transonic velocity potential equation to use as base for linear perturbation. Properties of particular solution, here given to second power about singular point of differential equation, are discussed elsewhere— report unobtainable by reviewer. Author seeks qualitative results applying far from body; obtains formally and by physical argument, for example, that change of pressure due to change of free-stream Mach number is proportional to $(M - 1)^{5/2}$ if stagnation pressure is kept constant. A. Charnes, USA

800. Crocco, L., Transforms of the hodograph flow equation and the introduction of two generalized potential functions, *NACA TN* 2432, 81 pp., Aug. 1951.

Paper consolidates and generalizes much of the hodograph investigation done thus far. Hodograph equations are symmetrized by using mass velocity and velocity as independent variables. A discussion of the approximate methods of Chaplygin and von Kármán-Tsien shows, in the light of the new symmetry, that a slight alteration of the von Kármán-Tsien method can probably be made to fit actual flows better. Exact solutions are given in terms of complex functions of streamline and potential functions and of the Legendre transformations of these. From the equations in these new functions, power-set solutions and exponential set solutions are obtained and discussed. Solutions by Lighthill of the power-set-type and by many authors using hypergeometric functions are shown to be special cases of the present formulation. Further, generalized complex potential functions of complex velocity and mass-velocity variables are introduced. The previously given functions and equations are given in terms of the generalized potentials. Solutions are obtained which may be

interpreted as exact solutions for the ideal gas law or approximate solutions for the actual gas law. A method of obtaining flows with circulation is given. Entire development is attractive because it is independent of the gas law, and because it appears capable of producing solutions valid in a small transonic region; but it does not, as yet, provide much new information on specific details of new sets of flows.   H. G. Cohen, Israel

801. Ghaffari, A. G., The hodograph method in gas dynamics, *Univ. Tehran, Fac. Sci. Publ.* **85**, 129 pp., 1950.

An introduction to the hodograph method is given from the mathematical point of view. The essential portion of the study is concerned with the velocity-distortion methods for purely subsonic flows; the methods of Temple and Yarwood, and Ghaffari are discussed in detail. Other chapters introduce hodograph equations, give properties of exact particular solutions, and give asymptotic representations for these solutions. A descriptive account of Bergman's operator method is also given.

Although the publication is not complete, the methods of von Kármán-Tsien and the exact methods of Cherry and Lighthill being omitted, it can be recommended as "an elementary survey of the theory and use of the hodograph method."

Hideo Yoshihara, USA

802. Drake, R. M., Jr., and Backer, G. H., Heat transfer from spheres to a rarefied gas in supersonic flow, Ann. Meeting ASME, Atlantic City, 1951. Paper no. 51—A-55, 9 pp.

Forced convection heat-transfer measurements are presented for spheres in air in the transition flow region (where gas density is so low that molecular mean free path is not negligible compared to sphere diameter). Data, covering Mach numbers from 2.28 to 3.56 and Reynolds numbers from 16 to 980, are correlated by plotting Nusselt number as function of (Reynolds number)$^{1/2}$ (Mach number). Authors' approximate analysis predicts dependence on above group. Reynolds number range is obtained mainly by changing gas density and sphere diameter (0.10 to 0.50 inch). Temperature recovery factors are also measured, verifying prediction that, at low Reynolds numbers, average equilibrium temperature of sphere increases above stagnation temperature of impinging air stream [see *J. aero. Sci.* **15**, 7, 381–391, July 1948].

Spheres were machined from silver, and single thermocouple was used to measure average sphere temperature. Heat-flow rates were obtained from temperature vs. time measurements in supersonic wind tunnel. Reviewer believes this paper is a significant addition to information available in a relatively unexplored flow region.   E. D. Kane, USA

803. Tchen, C.-M., Heat delivery in a compressible flow and applications to hot-wire anemometry, *NACA TN* 2436, 63 pp., Aug. 1951.

The problem of heat delivery by a heated body in a non-adiabatic rotational flow field is treated. By assuming that the variation of temperature along the stream is smaller than that along the transverse direction, author is able to relate total temperature and a function characterizing the rotationality of flow. With this relation, problem is reduced to solving two equations for the "generalized velocity potential" and temperature.

In an idealized case of a heated flat plate in a uniform stream, the problem is linearized for both subsonic and supersonic speeds. By establishing a solution of heat source, the source strength of continuous distribution along a straight line parallel to the stream corresponding to a given temperature distribution, and also the total heat delivery by such distribution, are calculated. From the form of equation for "generalized velocity potential," induced velocity by heating can be similarly expressed in terms of source distribution. In all these steps, approximations are introduced, sometimes tacitly.

Finally, author tries to extend method to blunt bodies by introducing further approximations which are subject to severe restrictions. Moreover, the neglect of the viscosity is discussed. It is shown that, for small perturbations and Prandtl number one, the case considered is justified, at least to first order.

Y. H. Kuo, USA

804. Spalding, D. B., Critical flow through convergent nozzles, *Aircr. Engng.* **23**, 270, p. 238, Aug. 1951.

Spalding takes exception to the assumption of the polytropic expansion law used by Naylor [see AMR 4, Rev. 4233] and attempts to establish that the throat velocity is always the local sonic velocity and that it is dependent on the inlet conditions and not on the friction characteristics. The expansion law in the throat is also independent of friction and, on the enthalpy-entropy chart, the expansion line is vertical at the point corresponding to the throat.

In his reply to Spalding (in the same issue) Naylor emphasizes that the expansion law which he assumed is in agreement with experimental results.

In the same issue Stephenson points out that the main conclusion of Naylor's original article was noted in the field of acoustics by Helmholtz in 1860, extended by Kirchhoff in 1868, and verified by Kaye and Sherratt in 1933.

Ione D. V. Faro, USA

805. Kestin, J., and Owczarek, J. A., Critical flow through convergent-divergent nozzles, *Aircr. Engng.* **23**, 272, 305–307, Oct. 1951.

In reply to Spalding's letter (of the August issue), the authors disagree with his assertions and give an analysis of the conditions under which sonic velocity may be obtained in or near the throat of a converging nozzle for the case of flow with friction present. In commenting on Naylor's original paper they refute one of the generalized conclusions that he makes in reference to the nature of the propagation of disturbances. They also show that, though the assumption of constant small stage efficiency is sufficiently accurate for approximation to the actual conditions, it can be shown that it does not reduce to a Fanno line in the case of constant area flow; further, it is inconsistent with the accepted one-dimensional equation of motion.   Ione D. V. Faro, USA

806. Naylor, V. D., Flow of a compressible gas with friction, *Aircr. Engng.* **23**, 272, 308–310, Oct. 1951.

Frictional compressible flow is considered by writing the energy equation in the form $-Vdp = \eta_\infty d(1/2 \ q^2)$, where η_∞ is a fractional factor introduced to account for only a partial transformation of the kinetic energy into pressure and thermal energy. (This factor is unity for isentropic flow.) Without considering the mechanism of the frictional process, as is done in Navier-Stokes equation, expressions for polytropic gas equation, sound velocity, and potential flow, are obtained by considering η_∞ to be a constant. These expressions show qualitatively how friction modifies the flow process.   Ray C. Makino, USA

807. Valensi, J., and Pruden, F. W., Some observations on sharp-nosed profiles at supersonic speed, *Aero. Res. Counc. Lond. Rep. Mem.* 2482, 17 pp., May 1947, published 1951.

Two-dimensional tests have been made on two airfoils, one a wedge section and the other a double wedge (rhombus) section, each with a nose semi-angle of 3.6 deg. Tests were made in a 5×2-in. wind tunnel at a Mach number of 1.4, and comprised

schlieren observation and pressure measurement. Results have been compared with exact inviscid theory. Several divergences were noticed and are discussed. Excellent agreement with theory was obtained under certain conditions, and where serious disagreement occurred it was partly attributable to wind-tunnel interference.

A satisfactory schlieren apparatus has been developed of sufficient sensitivity for observation at atmospheric stagnation density and moderate supersonic Mach numbers.

A number of schlieren pictures are given, some of which include a preliminary attempt to show up the filament lines of the flow.

From authors' summary

808. Goodman, Th. R., Aerodynamics of a supersonic rectangular wing striking a sharp-edged gust, *J. aero. Sci.* **18**, 8, 519–526, Aug. 1951.

Author briefly reviews Gardner's method [AMR **4**, Rev. 2124] for solving linearized three-dimensional nonsteady-wing problems. This method reduces a nonsteady finite wing problem to the equivalent of two steady finite wing problems. Author applies the method to the calculation of the pressure distribution, lift, and pitching moment on a rectangular wing penetrating a sharp edge gust while traveling at supersonic speeds. The solutions for the lift and pitching moment have also been given recently by Miles [AMR **4**, Rev. 2128].   John R. Spreiter, USA

809. Imai, I., and Hasimoto, H., Application of the W.K.B. method to the flow of a compressible fluid, II, *J. Math. Phys.* **28**, 4, 205–214, Jan. 1950.

In a previous paper [see AMR **3**, Rev. 2725] one of the present authors expressed the fundamental equations of motion for a two-dimensional compressible fluid flow in terms of new variables suggested by the W.K.B. method, and obtained a new method of approximation for treating the subsonic flow past arbitrary profiles. In this paper, the method is applied to some typical cases of interest in order to test its usefulness.

From authors' summary

810. Greidanus, J. H., Elementary considerations on one-dimensional compressible flows (in Dutch), *Ingenieur* **63**, 28, L.39–L.47, July 1951.

Elementary lecture includes basic equations of one-dimensional compressible flow and straight shock waves. As an explanation of the entropy law forbidding rarefaction shocks, a perpetuum mobile of the second kind operating with such shocks is described.   R. Timman, Holland

811. Martin, M. H., Steady, rotational, plane flow of a gas, *Amer. J. Math.* **72**, 465–484, 1950.

Author discusses steady rotational gas flows in two dimensions in a curvilinear coordinate system formed by streamlines and isobars (lines of constant pressure). (Flows for which streamlines are isobars are excluded.) This leads to the system of differential equations: [1] $u_p - y_\psi$, $_{up} = x_\psi$, $uy_p - vx_p = 0$, $uy_\psi - vx_\psi = \rho^{-1}$, where x, y are the Cartesian coordinates, u, v the velocity components, ψ the stream function, p the pressure, and ρ the density. If $\partial(x, y)/\partial(p, \psi) \neq 0$, a solution of [1] furnishes a solution of the Euler equation and the continuity equation. To obtain a determinate system one adds to [1] the equation of state: $\rho = f(p, S)$, where S is the entropy and specifies the entropy distribution by prescribing $S = S(\psi)$. By Bernoulli's theorem the speed q is a function of p and ψ. The flow is subsonic (supersonic) if $q_{pp} < 0$ ($q_{qq} > 0$), potential if $q_\psi \equiv 0$. If the Bernoulli function $q = q(p, \psi)$ is given, a flow can be obtained by solving the quasi-linear equation for the function

$$\theta = \theta(p, \psi) = \arctan (j/u)$$
$$q\{(q_{pp} - q\theta_p{}^2)/\theta_\psi\}_x + (q^2\theta_p)_p = 0 \qquad [2]$$

This excludes flows for which $\theta_\psi \equiv 0$ (isoclinic flow). Author derives the most general equation of state admitting isoclinic flows. If this equation is of the form $\rho = \Sigma(S)\Pi(p)$, then all isoclinic flows are either Prandtl-Meyer flows (that is, have a one-dimensional hodograph) or are obtained from such by a device due to Munk and Prim [*Nav. Ord. Lab. Rep.* NOLM 9281, 1947]. Finally, flows with $\theta = \theta(\psi)$ (rectilinear flows) are examined in detail.   L. Bers, USA

812. Saito, O., and Amemiya, A., On the solution of differential equations of the two-dimensional steady flow of compressible fluid, *J. phys. Soc. Japan* **5**, 201–202, 1950.

In a potential gas flow the density ρ is a given function of the speed q. Set

$$w = \int \frac{\rho dq}{q}, \ f(w) = \frac{1}{\rho^2} - \frac{d}{dw}\left(\frac{1}{\rho}\right)$$

If $F(w, \psi)$ satisfies the equation

$$F_{ww}F_{\psi\psi} - F^2{}_{w\psi} = f(w) \qquad [1]$$

then $\phi = F_w$, $\theta = -F_\psi$ may be interpreted as the potential and inclination of the velocity vector, respectively, of a gas flow with stream-function ψ. Solutions of [1] are:

$$F = \psi \int (-f)^{1/2} dw + \alpha F + G(w)$$

where α is an arbitrary constant, G an arbitrary function,

$$F = (\psi + C)^2(Aw + B)^{-1} + \frac{1}{2}\int dw \int (Aw + B)f dw$$

where A, B, C are arbitrary constants. A detailed presentation will appear elsewhere.   L. Bers, USA

813. Krasil'shchikova, E. A., Pressure distribution on a lifting surface (in Russian), *Doklady Akad. Nauk SSSR (N.S.)* **79**, 5, 747–750, Aug. 1951.

Expressions in closed form are presented for the pressure distribution on an arbitrarily deformed lifting surface of arbitrary shape in uniform supersonic flow (without derivation). Contour integrals over the leading edge and sides are used in addition to surface integrals over various regions of the surface. The results are similar to those given by Evvard [*Nat. adv. Comm. Aero. tech. Notes* 1382, 1484, 1585] and others.   F. W. Diederich, USA

814. Bursnall, W. J., and Loftin, L. K., Jr., Experimental investigation of the pressure distribution about a yawed circular cylinder in the critical Reynolds number range, *NACA TN* 2463, 34 pp., Sept. 1951.

An experimental investigation has been made of the pressure distribution about a circular cylinder at yaw angles of 0°, 15°, 30°, 45°, and 60° for Reynolds numbers from below the critical value up to about 5.0×10^5. The Reynolds number is based on the cylinder diameter and the component of velocity normal to the leading edge of the cylinder. The Mach number of the flow normal to the cylinder was less than 0.2 for all tests. The results of the investigation indicated that, for the range of Reynolds number near and above the critical value, the flow and force characteristics of a yawed circular cylinder cannot be determined only by the component of flow normal to the axis of the cylinder. For example, the critical Reynolds number decreased from 3.65×10^1 for the unyawed cylinder to 1.00×10^5 for the 60° yawed

cylinder, and the supercritical drag coefficient, based on the flow normal to the leading edge of the cylinder, increased from approximately 0.18 for 0° yaw to approximately 0.74 for 60° yaw. In addition, the localized regions of laminar separation that appeared in the supercritical range of Reynolds number on the unyawed cylinder were not as well defined at yaw angles of 15° and 30° and completely disappeared at yaw angles of 45° and 60°.

From authors' summary

816. **Wang, C.-T., and de los Santos, S., Approximate solutions of compressible flows past bodies of revolution by variational method,** *J. appl. Mech.* **18**, 3, 260–266, Sept. 1951.

Direct method of Rayleigh-Ritz is applied to steady, inviscid, irrotational, compressible flow past bodies of revolution. Special care is necessary as the variational integral cannot be used directly if the domain is infinite. Two examples, viz., of a sphere and of an ellipsoid of revolution, are considered in detail. Excellent agreement with results of other investigators is obtained in these cases.   A. van Heemert, Holland

816. **Neice, S. E., and Ehret, D. M., Similarity laws for slender bodies of revolution in hypersonic flows,** *J. aero. Sci.* **18**, 8, 527–530, 568, Aug. 1951.

The hypersonic similarity law, as developed by Tsien for bodies of revolution at zero angle of attack, is tested by comparing pressure distributions obtained using the method of characteristics. The results indicate that, in spite of the assumptions made in its development, the hypersonic similarity law is valid for relatively wide ranges of Mach number and fineness ratio. Similarity laws for hypersonic flow about slender pointed bodies of revolution at angle of attack are also developed. Both the potential flow and the viscous effects of the transverse component of the flow are considered in this development.

From authors' summary by Chieh-Chien Chang, USA

817. **Lees, L., Note on the hypersonic similarity law for an unyawed cone,** *J. aero. Sci.* **18**, 10, 700–702, Oct. 1951.

It is now known that the hypersonic similarity law derived for slender cones and ogival bodies under the assumption $M \gg 1$, is applicable for Mach numbers as low as 3. This note makes use of a series development to infer the hypersonic similarity law for unyawed cones from the Taylor-Maccoll differential equations and associated boundary conditions. A simple approximate formula for the function $F(K) = C_D M_1^2$ of the similarity law is obtained, and the drag function computed with this formula is compared with Kopal's numerical results and, for very slender cones, with von Kármán's linearized formula.   R. C. Prim, III, USA

818. **Wegener, P., Reed, S., Jr., Stollenwerk, E., and Lundquist, G., Air condensation in hypersonic flow,** *J. appl. Phys.* **22**, 8, 1077–1083, Aug. 1951.

"Hypersonic" here refers to Mach numbers greater than about 5, with mean free path negligibly small compared with boundary-layer thickness. Experiments in a 12-cm square tunnel showed that air condensation occurred near or shortly after calculated dew line. Authors suggest that absence of considerable supersaturation may be due to presence of "foreign" nuclei, such as carbon dioxide, or ice particles. With a pure gas such as nitrogen, much greater supersaturation might be expected.

Measurements show that condensation has more effect on static pressures and shock-wave angles on slender bodies than on pitot pressures. Mach numbers calculated from static pressures agree with those found from Pitot pressures, even for stagnation temperatures that are not high enough to avoid condensation. This suggests that for some wind-tunnel work it may not be essential to avoid condensation completely.

Paper concludes with simple discussion of thermodynamics of condensation.   W. A. Mair, England

819. **Stalder, J. R., Goodwin, G., and Creager, M. O., Heat transfer to bodies in a high-speed rarefied-gas stream,** *NACA TN* 2438, 25 pp., Aug. 1951 = Gener. discuss. heat transfer Lond. Conf., Sept. 11–13, 1951, Sec. II. London, Inst. mech. Engrs.; New York, Amer. Soc. mech. Engrs., 7 pp.

A low-density supersonic wind tunnel, operating at jet pressures from 80 to 200 microns of mercury absolute, was utilized to determine the heat transfer to cylindrical bodies placed normal to the jet stream. Special ventilated nozzles were employed to bleed off the boundary layer and give a uniform Mach number over the center third of the jet stream. The tunnel was operated at Mach numbers of 2.00, 2.50, and 3.15. The data are reported in terms of the Knudsen number, which is defined as the ratio of molecular mean free path to test body diameter. The cylindrical test bodies used were wires from 0.0010 to 0.126 in. in diam. The possible combinations of tunnel pressure and wire diameter gave a range of Knudsen numbers from 0.025 to 11.8. The associated Reynolds numbers varied from 0.28 to 203.

It was found that free molecular flow is completely developed at Knudsen numbers ≥ 2.0. The temperature-recovery factor depends primarily on the Knudsen number, exceeding unity for Knudsen numbers > 0.2. Over the range of the conditions in the tests, the Nusselt number was found to be a function of only the Reynolds number if the viscosity and thermal conductivity are based on stagnation temperature and the density is based on free-stream conditions. Experimental work verified the theoretical prediction that in free-molecular flow, the insulated body equilibrium temperature would exceed the stagnation temperature of the gas stream.   Thomas P. Clark, USA

820. **Kämmerer, C., Steady gas flow through a straight pipe with and without heat transfer and friction** (in German), *Öst. Ing.-Arch.* **5**, 4, 340–370, 1951.

Author, starting from the four well-known equations for one-dimensional pipe flow of an ideal gas (energy, impulse, continuity, state), in which friction is introduced by $dh_{fr} = -\frac{1}{2}\rho w^2 \lambda dl/D$ (dh_{fr} is loss of head in friction, ρ density, w velocity, l pipe length, D diameter, λ friction factor), considers the following cases with $D =$ constant and $\lambda =$ constant: (1) $dh_{fr} = 0$; (2) $dq/dT =$ constant (T is static temperature, q heat added); (3) $dq/dl =$ constant; (4) $\beta = k(T_a - T)$ with constant k and T_a: $\beta = dq/dl$; (5) $\rho =$ constant; (6) $p = \rho^n \cdot$constant for $n = \gamma, 1, 0, -0.738, -1$ ($\gamma = c_p/c_v =$ ratio of specific heats).

Most cases are dealt with comprehensively; a number of diagrams explain results of calculation. Validity in practice of formulas obtained is discussed.   H. Wijker, Holland

821. **Lomax, H., and Heaslet, M. A., Generalized conical-flow fields in supersonic theory,** *NACA TN* 2497, 44 pp., Sept. 1951.

A conical field of order n is a field of supersonic flow in which the induced velocity components along radial lines through a fixed vertex are proportional to the nth power of the distance from the vertex. Authors consider problem of calculation of pressure distribution on specified yawed or unyawed delta wings with subsonic leading edges, and problem of calculation of shape for given pressure distribution. Method is based on potentials due to particular source distributions along radial lines through vertex. For conical fields of order n, solutions under consideration are obtained as linear combinations of known functions with constant coefficients which can be determined by solving a set of linear equations. Apart from known cases such as rolling and pitching

delta wings, authors calculate, for example, the effect of parabolic twist. Concerning problems of the second class, authors obtain remarkable result that various wing shapes are possible for given pressure distribution.

Extensive use is made of Hadamard's "finite part of an infinite integral." Although more general methods to cope with infinities arising in such problems are now available (methods of M. Riesz and L. Schwartz), reviewer agrees that for many practical problems Hadamard's approach possesses advantage of relative simplicity. It is pointed out that the concept called "generalized finite part" by present authors has been used already by Hadamard and others.　　　　　　　　　　　　A. Robinson, Canada

822. Oswatitsch, K., The effect of compressibility on the flow around slender bodies of revolution, *Roy. Inst. Technol., Div. Aero., Stockholm, KTH-Aero TN* 12, 8 pp., 1950.

Author derives simple logarithmic relation between Mach numbers and change of velocity distribution in subsonic and supersonic flow valid for potentials given in usual manner by distribution of body's cross-sectional area. Total pressure drag minus base pressure drag is independent of Mach number in supersonic flow if body has cylindrical rear end.

From author's summary by A. Charnes, USA

823. Schultz-Piszachich, W., Contribution to the calculation by formulas of the stationary velocity distribution around bodies of revolution in subsonic and supersonic flow (in German), *Öst. Ing.-Arch.* 5, 4, 289–303, 1951.

A method, based on linear theory, is presented for calculating the velocity distribution around slender bodies of revolution in both axial and inclined flows. The development proceeds essentially through a standard separation-of-variables process and culminates in each case with an infinite series representation for the velocity components. Author is apparently unaware of the extensive literature that has been built up on this subject in recent years; the most recent reference is dated 1943.

John R. Spreiter, USA

Turbulence, Boundary Layer, etc.

(See also Revs. 572, 937)

824. Davies, D. R., A note on three-dimensional turbulence and evaporation in the lower atmosphere, *Proc. roy. Soc. Lond. (A)* 202, 1068, 96–103, June 1950.

Under the assumption that the lateral diffusivity varies as the same power of height as the wind velocity, author obtained in a previous paper [title source 190, p. 232, 1947] a formula for computing the vapor concentration over a parabolic evaporation area. In present paper author transforms that formula into a simpler and more significant form, compares the result with that obtained by Calder [AMR 3, Rev. 2015] for two-dimensional diffusion, and with experimental figures obtained in England. At points near the axis of the evaporation area, theoretical and experimental figures at various height sometimes differ by as much as 75%, but show good agreement in order of magnitude. The agreement is less satisfactory at larger distances from the axis of the evaporation area. In concluding discussion, author implies that the agreement between theory and experiment is sufficiently good for the hypothetical law of lateral diffusivity to be considered verified or at least probable.　　　Chia-Shun Yih, France

825. Tamaki, H., On the solution of the laminar boundary-layer equations (in Japanese), *Rep. Inst. Sci. Technol. Tokyo,* 5, 1/2, 49–62, Aug. 1951.

Author develops an approximate method of solving the equations of steady two-dimensional laminar boundary layer by using the Mises transformation.

For incompressible fluids, the equation of motion is

$$\partial Z / \partial \varphi = \nu (u/u_1) \varphi^2 Z / \varphi \psi^2,$$

where $2Z = u_1^2 - u^2$, $\varphi = {}_0\int^x u_1 dx$, ν is kinematic viscosity, u velocity in the boundary layer, u_1 outside velocity, x distance along the surface, and ψ stream function. The essential point of author's solution is that u/u_1 on the right side of equation is substituted by von Kármán-Millikan's "outer solution," and that only the first term of the series in ψ is retained as approximation. Then, changing independent variables, Z is obtained in an integral form, distribution of u_1 being contained in the integrand. The approximation seems to be good enough; the method, when applied to the flow for which exact solution is known, leads to satisfactory results.

Author also considers compressible fluid with Prandtl number unity under the condition of zero heat transfer from the surface. By suitable modifications in the definitions of φ, ψ and Z, the equation is reduced to the same form as for incompressible fluids. Similarity between compressible and incompressible fluids is discussed.　　　　　　　　　　　　Ichiro Tani, Japan

826. Laurmann, J. A., Stability of the compressible laminar boundary layer with an external pressure gradient, *Coll. Aero. Cranfield Rep.* 48, 64 pp., 12 figs., Sept. 1951.

Author applies the original approach by Lin and Lees to the stability problem of compressible laminar boundary layer to a boundary with an external pressure gradient. He comes to the conclusion that, in essentials, the theory developed by Lees and Lin is applicable to flows with an external pressure gradient, and that the stability again depends on the local velocity and temperature profiles of the boundary layer only. In both cases there is a Reynolds number based upon the boundary-layer thickness above which the laminar flow is inherently unstable and below which the boundary layer is completely stable. To obtain the compressible boundary-layer velocity distribution, author applies some elementary transformation to the known flow solutions calculated by Hartree for the incompressible boundary layer. The numerical analysis follows that of Lees and Lin very closely. Results up to $M \sim 3$ are given in forms of diagrams.　　　　　　　　　　　M. Z. Krzywoblocki, USA

827. Kampé de Fériet, J., and Betchov, R., Theoretical and experimental averages of turbulent functions, *Proc. kon. Ned. Akad. Wet. (B)* 54, 4, 389–398, Sept./Oct. 1950.

Paper discusses four axioms which must be assumed valid in computing mean values by method of Reynolds: (1) $\overline{f + g} = \overline{f} + \overline{g}$; (2) $\overline{af} = a\overline{f}$ where a is a constant; (3) $\overline{fg} = \overline{f}\overline{g}$; and (4) $\operatorname{Lim} \overline{f_n} = \operatorname{Lim} f_n$ for some convenient definition of the limiting process. All four axioms must be satisfied to make Reynolds' equations logical consequence of Navier-Stokes equations. Generally, all except third are satisfied, and it is satisfied approximately if the energy is principally due to large wave numbers. Paper has applications in turbulence measurements, where the mean values of some function $f(t)$ are obtained by a low-pass filter or equivalent.　　　　Louis M. Laushey, USA

828. Moore, F. K., Unsteady laminar boundary-layer flow, *NACA TN* 2471, 33 pp., Sept. 1951.

Theoretical study of the compressible laminar flow over a semi-infinite flat plate moving in a straight path with a time dependent speed. Solution is given with the Prandtl boundary-layer assumptions for constant acceleration and an arbitrary but dif-

ferentiable velocity-time relation. Author introduces dimensionless parameters, which represent a measure for the promptness with which the boundary layer responds to impress changes. If these parameters are small, nearly quasi-steady flow is to be expected; i.e., the flow corresponds to the steady flow calculated with the conditions prevailing at that instant. Numerical calculations of the deviation of the velocity and temperature profiles from the quasi-steady flow of an insulated plate are given. For very large parameters, the classical "starting from rest" solution holds.

Generalizing his analysis of the unsteady flow over a flat plate with constant pressure distribution, author recommends the use of similar parameters as an advance criterion, whether a given unsteady laminar flow with pressure gradient, or even the case of turbulent boundary layers have to be treated either as quasi-steady or not. As an example, he considers the flow over a flat plate with sinusoidal oscillating stream velocity.

Karl Pohlhausen, USA

829. Rotta, J., Statistical theory of nonhomogeneous turbulence (in German), Z. Phys. **129**, 6, 547–572, 1951.

Paper deals with the statistical theory of inhomogeneous turbulence in shear flow with an approach and analysis similar in many ways to that used previously by P. Y. Chou [Quart. appl. Math. 3, 38–54, 1945; compare Eqs. (2.10), (2.17), (3.9) of the present paper with Eqs. (1.6), (2.1), (5.1) in Chou's paper]. Differential equations for the change of Reynolds stresses and kinetic energy are derived. The various terms occurring in these equations, corresponding to turbulent diffusion and other mechanisms, are then analyzed and simplified by physical reasoning and other approximations. In particular, the relation (4.10) is obtained with the help of modern concepts of turbulence and differs from Eq. (6.14) of Chou's paper. After such simplifications, the theory is applied to calculate the statistical averages of various quantities for two-dimensional channel flow and compared with the experimental results of Richardt and Laufer in a rectangular channel.

C. C. Lin, USA

830. Kurihara, M., The turbulent field in a wake behind a symmetrical cylinder, Rep. Res. Inst. Fluid Engng. Kyushu Univ. 6, 2, 60–66, Jan. 1950.

Diffusion of turbulence energy and of momentum are considered in the wake behind a symmetrical cylinder. With the usual approximations the equation of motion for the direction of flow reads: $U_0 \partial u/\partial x = \partial/\partial y(\kappa \partial u/\partial y)$ (U_0 is free-stream velocity, u velocity defect, κ eddy viscosity, x and y coordinates).

If effect of pressure fluctuations (strictly speaking not correct) is neglected, and a coefficient of eddy diffusion equal to κ introduced, the diffusion of turbulence energy $c^2 = u'^2 + v'^2 + w'^2$ (u', v', w' turbulence components) is given by

$$U_0 \partial c^2/\partial x = 2\kappa(\partial u/\partial y)^2 + \partial/\partial y (\kappa \partial c^2/\partial y) - c^4/12\kappa$$

The right-hand terms originate, respectively, from the work by shear stresses, from eddy diffusion and from dissipation ($= 15 \nu u'^2/\lambda^2$, ν kinematic viscosity, λ micro scale).

It is argued that $\kappa = \sim$ constant across the wake. In that case the velocity defect is Gaussian: $u = u_0 \exp(-y^2 U_0/4\kappa x)$. Author now assumes that the distribution of c is also Gaussian: $c = c_0 \exp(-y^2 U_0/4\alpha x)$ He then obtains, after some calculations from the diffusion equation, the interesting result that the width of the turbulent field is always greater than that of the velocity defect because $(\alpha/\kappa)^{1/2}$ is found greater than one. It further turns out that this ratio can only vary between the limits $2/(2)^{1/2} = 1.68$ and $4/(2)^{1/2} = 2.83$, and that at the same time c_0/u_0 varies from infinity to zero, whereas $(u_0/U_0) (U_0 x/\kappa)^{1/2}$

varies from zero to infinity. In a graph, c_0/u_0 and $(\alpha/\kappa)^{1/2}$ are plotted against $(u_0/U_0)(U_0 x/\kappa)^{1/2}$.

Author's interpretation is that c and κ increase with increasing Reynolds number, so that large values of c_0/u_0, and at the same time small values of $(u_0/U_0)(U_0 x/\kappa)^{1/2}$ must occur at large Reynolds numbers, and vice versa.

Comparison with experimental data by Dryden and Kuethe [NACA Rep. 342, 1930] and by Fage [Aero. Res. Counc. Lond. tech. Rep. 1520, 1932] proves satisfactory.

J. O. Hinze, Holland

831. Jones, Sir Melville, and Head, M. R., The reduction of drag by distributed suction, 3rd Anglo-Amer. aero. Conf., Brighton, Sept. 4–7, 1951. Roy. aero. soc., 199–230.

Paper describes experiments (in flight) performed at the University of Cambridge, England. Paper is in three parts: (1) Brief discussion of the causes of transition to turbulence in the boundary layer both in the absence and presence of distributed suction; (2) description of Cambridge experiments and comparison of results with those obtained at Langley Field; (3) estimate of the equivalent profile drag for an ideal airplane over which laminar flow is maintained over its entire surface by distributed suction.

The idea of distributed suction is to increase the stability of the boundary layer by modifying the velocity profiles to a more stable form even in the presence of pressure gradients and by keeping the boundary layer thin to any desired extent. The feasibility of this method is indicated by theoretical calculations of the boundary layer over a porous flat plate, showing that there is a critical value of suction velocity above which the flow never becomes unstable ($v_s/U \sim 1.5 \times 10^{-4}$, v_s suction velocity, U free stream velocity).

Apparatus for the experiments consists of a model airfoil with a porous surface incorporated in one side. Porous surface is made of calendered nylon fabric supported, in some experiments, on tightly stretched wire gauze supported, in turn, on closely spaced ribs. In later experiments where the suction flow was small, the nylon fabric was backed by filter paper and supported on perforated zinc. Model wing was then attached below the fuselage (with its span vertical) of an Anson Mk. I.

Early experiments on this model wing in flight indicated that small surface imperfections had a detrimental effect on the reduction of drag by means of suction. It appears that once transition had occurred due to the surface imperfections, no amount of suction that was available was sufficient to restore laminar flow. Conclusions from these and later experiments indicate that: (1) On a smooth porous surface of any length over which the pressure and the suction velocity are uniform, laminar flow can be maintained by a suction velocity which is no greater than that required to preserve the stability of the layer and may even be less than this. (2) If laminar flow is maintained, the velocity profiles of the boundary layer agree well with those predicted theoretically (by Iglisch, exact calculations for uniform suction over a flat plate). (3) These conclusions are not invalidated if the surface has a slight convex curvature or if it is nonporous near the leading edge.

Some experiments were also made on the effect of surface roughness in causing turbulent flow over surfaces with distributed suction. Results indicate that surface roughness has a large influence in preventing laminar flow with any amount of suction above a certain speed of the wing, and that for laminar flow the surface in the areas near the front of a body will have to be very smooth and free from flies or any small obstruction of the flow.

Measured velocity profiles in the presence of pressure gradients with suction are of a form similar to those found in the absence of pressure gradients. Theoretical calculations indicate that the

suction ratios required to preserve stability of the boundary layer fall rapidly with increase of Reynolds number in the presence of pressure gradients. A few experimental points also indicate this behavior. Calculations are presented for the equivalent profile drag of airfoils with distributed suctions. (Equivalent drag is that drag force which could give rise to the same consumption of power for propulsion as is required to overcome the net drag and to energize the air removed by suction.) These calculations indicate it is possible to reduce this drag to values very much smaller than could possibly be achieved without suction or any other method of boundary-layer control. Comparison of profile drag and equivalent profile drag of experimental values at Cambridge with Langley and theory is good.

On these bases it appears there is a possibility of maintaining laminar flow by distributed suction applied over almost the whole surface of an airplane since the adverse pressure gradients anywhere on the plane will not exceed that on the wing. Approximate calculations indicate that the profile drag of this ideal aircraft, with account taken of the power required to energize the air removed by suction, would be less than one-tenth of that of the best contemporary aircraft.

Reviewer believes the results show great promise and agrees with the authors that many theoretical and practical problems will have to be solved before laminar flow can be reliably maintained by distributed suction. Reviewer also thinks that further applications appear in its use to control the location of shock waves and reduction of drag on supersonic aircraft.

Henry G. Lew, USA

832. Ferrari, C., The turbulent boundary layer in a compressible fluid with positive pressure gradient, *J. aero. Sci.* **18,** 7, 460–477, July 1951.

For a turbulent boundary layer in a compressible flow with a positive pressure gradient, the following features are investigated: velocity distribution, thickness of the layer, drag coefficient, and separation. Theory is compared with experiments of Gruschwitz and Kehl at low speed. From an assumption concerning the dependence of the shearing stress and the characteristics of the mean flow, the equation of motion for the turbulent boundary layer is reduced to the same form as that used by von Mises for the laminar boundary layer. The solution of the equation is then obtained by using the method of von Kármán.

C. M. Tchen, USA

833. Ferrari, C., Comparison of theoretical and experimental results for the turbulent boundary layer in supersonic flow along at flat plate, *J. aero. Sci.* **18,** 8, 555–564, Aug. 1951.

On the basis of experimental results [Young, E. C., and Wilson, R. E., Bumblebee CF-1095, DRL, no. 181, Sept. 1948], author extends previously obtained theoretical data up to $M = 2$, [AMR 4, Rev. 1293 and preceding review]. Earlier theory is now restricted to an "inner" region, while different formulas are applied to an "outer" region. Thus better agreement with experiments is obtained.

Author uses successive approximations in predicting velocity profiles and fiction coefficient. Theoretical results show a decrease in drag coefficient with increasing Mach number. Some diagrams and tables give theoretical results and corresponding experimental data.   Friedrich Keune, Sweden

834. Szablewski, W., Calculation of the turbulent flow along a plane plate (in German), *ZAMM* **31,** 10, 309–324, Oct. 1951.

Applying Prandtl's assumption for the shearing stresses, author integrates the differential equation for the flow of a viscous fluid along a flat plate. The integration starts from the wall

and is performed in two steps. The first covers the laminar sublayer, the annexing range of equal magnitude of laminar and turbulent friction and that part of predominant turbulent friction where a linear mixing length distribution can be assumed. The velocity distribution for this step is characterized by a logarithmic law. For the second step, the laminar friction can be neglected, but the inertia terms have to be taken into consideration. The solution obtained represents an approximation for the remaining part of the boundary layer and can be improved by an iteration process. Finally, the thickness of the boundary layer and the drag coefficient are calculated as function of the Reynolds number. The theoretical results are in good agreement with experimental values if the empirical coefficients entering in the theory are chosen conveniently.   Karl Pohlhausen, USA

835. Rubert, K. F., and Persh, J., A procedure for calculating the development of turbulent boundary layers under the influence of adverse pressure gradients, *NACA TN* 2478, 61 pp., Sept. 1951.

Authors' analysis of incompressible turbulent boundary-layer flow is an attempt to secure improved prediction of characteristics of diffuser flows. A modification of the von Kármán momentum equation is made by including the turbulent-normal-stress coefficient. The Tetervin-Lin energy equation, which is a particular integral form of Prandtl's boundary-layer equation, is used as an auxiliary relation to determine the shape parameter H. Empirical correlations of the wall stress, the turbulent-normal-stress, and the dissipation coefficients in terms of the pressure gradient, shape factor, and boundary-layer Reynolds number are offered. Authors emphasize that these correlations are of an interim nature and may be expected to be changed with advent of new data covering a broader range. Comparisons of experimental results with predictions by method given are made for flow over flat plates, airfoils, and in conical diffusers. Short of separation, the agreement is good in most cases and appears sufficient to justify refinement of the analysis. Perhaps it is because of the closeness of publication dates that no mention is made of Granville's proposed solution [AMR 4, Rev. 3946] which uses another particular Tetervin-Lin integral form of the boundary-layer equation to establish the variation of H.   John P. Breslin, USA

Aerodynamics of Flight; Wind Forces

(See also Revs. 619, 673, 770, 779, 807)

836. Milliken, W., Jr., Dynamic stability and control research, 3rd Anglo-Amer. aero. Conf., Brighton. Sept. 4–7, 1951. Roy. aero. soc., 447–524.

This is author's second paper of this type [AMR 1, Rev. 154] and in both he has done a good job of surveying a large and important field of aeronautical endeavor. The discussion of any one topic considered may not be sufficiently detailed to satisfy a specialist in that field, but this is compensated for by the inclusion of 127 references.

Author restricts his treatment for the most part to the dynamics of the airframe, although he discusses the importance of the relationship between the airframe and the oft-employed automatic control devices. Moreover, only the developments in dynamic stability and control research that have taken place in America are reviewed. The following nine topics were selected for discussion because of their relationship to present design difficulties: Full-scale dynamic response measurements, nonstationary flow effects, higher frequency response measurements, problems of response measurements at transonic and supersonic speeds, prediction of dynamic characteristics, dynamics of

the stall, phugoid and spiral modes, artificial stability and rational specification determination, and gust response measurements.

Arthur L. Jones, USA

837. Bollay, W., Aerodynamic stability and automatic control, *J. aero. Sci.* **18**, 9, 569–617, Sept. 1951.

Paper is text of the fourteenth Wright Brothers Lecture. Author presents past, present, and future of automatic control for aircraft, with particular emphasis on methods of analysis. History of automatic control in aircraft is presented, with statement of some outstanding problems confronting designer. Detailed consideration is given to present-day methods of analysis of closed-loop automatic control systems. The use of the root-locus system and the Routh criteria are explained in detail with illustrations. Future application of automatic control to aircraft is proposed.

Reviewer finds paper an unusually clear and complete presentation of the subject. It is believed that for many years to come this paper will be a fundamental reference for those interested in methods of analysis of automatic control systems.

Wilbur L. Mitchell, USA

838. Campbell, J. P., Hunter, P. A., Hewes, D. E., and Whitten, J. B., Flight investigation of the effect of control centering springs on the apparent spiral stability of a personal-owner airplane, *NACA TN* 2413, 46 pp., July 1951.

839. Harper, J. J., Wind-tunnel investigation of effects of various aerodynamic balance shapes and sweepback on control-surface characteristics of semispan tail surfaces with NACA 0009, 0015, 66-009, 66(215)-014, and circular-arc airfoil sections, *NACA TN* 2495, 127 pp., Oct. 1951.

Author presents a wealth of useful data on tail control-surface characteristics measured at low speeds in 8-ft tunnel at Georgia Tech. Section on prediction of aerodynamic parameters, however, is somewhat superficial. References, for instance, do not include most up-to-date summary on prediction of tail-surface aerodynamic parameters [*NACA TN* 2288; AMR **4**, 3662], which presents more satisfactory formulas.

Arthur L. Jones, USA

840. Becker, L., Control-surface position feedback in flight-control systems, *Aero. Engng. Rev.* **10**, 9, 17–19, 33, Sept. 1951.

Advantages and disadvantages of position feedback in autopilots are discussed with clarity and conciseness. Discussion is qualitative with a minimum of mathematics. Advantages pointed out are improved dynamics, a more linear system, and less dependence of system characteristics on surface hinge moments. Possible disadvantages mentioned are high frequency instabilities, loss of static sensitivity, and need for automatic tuning of autopilot for changing flight conditions.

Usually these disadvantages can be overcome by accepted engineering techniques, although complicated by increased gadgetry.

With the ever-increasing performance demands of autopilots, reviewer believes position feedback and its inherent complications are essential for any but the simplest of control systems. The general trend in the field is toward even more complicated systems.

Graham Campbell, USA

841. Carline, A. J. K., Still-air range in the stratosphere. A method of rapid and accurate calculation for jet-propelled aeroplanes operating above the tropopause, *Aircr. Engng.* **23**, 272, 292–299, Oct. 1951.

Paper deals with an analysis of the problem of calculating the range of aircraft at high altitudes. The assumption of flight in the constant temperature region of the stratosphere along with the further assumptions of constant true air speed and lift coefficient lead, in the case of jet-propelled aircraft, to operation at a constant value of the ratio of the gross weight (or thrust) to the relative density. On this basis the integration of the range equation is somewhat simplified.

Considerable attention is given to the development of a simplified approximate expression for range based on use of the mean values of thrust and weight during the flight. These approximations are found to be quite accurate for usual values of ratio of fuel load to empty weight. There is also a good deal of analysis concerned with effect on range of small changes in the various design parameters of the aircraft.

No consideration is given in the paper to the estimation of the initial portion of the range corresponding to the climbing flight into the stratosphere. Also, while considerable attention is given to the determination of the height at which half the fuel is consumed and the height over the target (at half-range) in the case of bomber-type aircraft, the effect of reducing weight by dropping the bomb at the latter point is not considered.

M. J. Thompson, USA

842. Thompson, L. N., Fundamental dynamics of reaction-powered space vehicles, *Aircr. Engng.* **23**, 270, 228–234, Aug. 1951 = *Instn. mech. Engrs. Proc.* **164**, 3, 264–273, 1951.

Paper obtains the velocity conditions for escape from the earth's orbit and for extraterrestrial flights, and discusses the likely available energy sources for their achievement. In part, paper covers same ground as Malina and Summerfield [JLP /CIT Pub. no. 5, Aug. 1946, apparently unknown to author]. Theoretically, obtainable jet velocities of various chemical fuels are deduced, but these are high compared with known practical values, not discussed by author. He postulates a "thermodynamic type of atom rocket motor employing an inert reaction mass to absorb the fission energy," since he concludes that chemical energy is insufficient to achieve interplanetary travel.

R. C. Knight, England

843. Timman, R., The direct and the inverse problem of aerofoil theory. A method to obtain numerical solutions, *Nat. LuchtLab. Amsterdam Rap.* F.16, 30 pp., 1 table, 1951.

Paper summarizes various numerical methods used to solve either the direct problem of determining the two-dimensional, incompressible, potential pressure distribution about any given airfoil profile, or the inverse problem of deriving the airfoil profile which has a given pressure distribution.

Commonly known methods for the conformal mapping of an arbitrary profile onto a unit circle are reviewed. The convergence of the various numerical methods is discussed and a new iteration method is proposed, utilizing interpolation polynomials in conjunction with matrixes. The new method avoids the undesirable waviness resulting from the usual trigonometric approximations. Either the direct or inverse problem may be solved, and examples are worked out deriving the roof-type or so-called laminar-flow airfoils.

Reviewer notes that the first successful solution of the inverse problem, and still the simplest practical approximation, is by H. J. Allen: "General theory of airfoil sections having arbitrary shape or pressure distribution" [*NACA Rep.* 833].

E. V. Laitone, USA

844. Squire, H. B., Jet flow and its effects on aircraft, *Aircr. Engng.* **22**, 253, 62–67, Mar. 1950.

Experimental data of round jets in still air and in general

stream are first analyzed. Empirical formulas for the spread of the round jet of low subsonic flow and the decay of the axial velocity distribution are given. Similar formulas are given for the temperature distribution for jet of low subsonic flow with moderate temperature difference between the jet and the surrounding stream. The results for plane jets and the effect of supersonic flow are briefly mentioned.

The method of model test technique for the jets and jet aircraft are then discussed. It is shown that the jet momentum is the most important quality in the representation of hot jet.

The interaction between the jets and neighboring surfaces are then discussed, which includes the interference losses (negligibly small), temperature contours in oblique jet tests, the Coanda effect (that a plane jet emerging from a nozzle near a curved surface tends to adhere to the surface), jet impinging on the ground, and the coalescence of two round jets.

Finally, the effects of jet on aircraft stability, practically on the flow at the tailplane and on rotary derivatives, are briefly discussed.   S. I. Pai, USA

845. Pearson, H. A., McGowan, W. A., and Donegan, J. J., Horizontal tail loads in maneuvering flight, *NACA Rep.* 1007, 12 pp., 1951.

Report makes use of so-called "inverse" method. An airplane is assigned a normal-load-factor time history, using the specified maximum value and a time necessary to reach peak value, resulting from a triangular-impulse type elevator displacement. The duration of the elevator displacement is determined empirically from flight data for various airplane types. The tail load is separated into components involving linear functions of the airplane normal load factor and its derivatives. Simple approximate expressions are developed for the maximum up- and down-loads on the tail, which permit a very rapid estimation of these loads. Maximum pitching velocities and accelerations may also be determined simply. Examples are given to illustrate the method and to show the effect of the principal variables.   Gabriel Isakson, USA

846. Brown, R. B., Holtby, K. F., and Martin, H. C., superposition method for calculating the aeroelastic behavior of swept wings, *J. aero. Sci.* 18, 8, 531–542, Aug. 1951.

The method starts with the angle-of-attack distribution α_F of the deflected wing, determines the aerodynamic loading and from this the deflection-angle distribution α_E due to the bending and torsion of the elastic swept wing. Thus, the aerodynamic and structural problems are separated and can be solved by any known method. The corresponding angle-of-attack distribution α_I of the rigid wing is obtained from $\alpha_I = \alpha_F - \alpha_E$. This solution can be denoted as the "direct problem."

The more difficult "indirect problem" to find α_F for a given α_I-distribution can be reduced to the direct problem. For this purpose the direct problem, which is only dependent on the wing planform and structural design parameters, has to be solved for certain characteristic spanwise distributions of α_F. Both the aerodynamic loading and the structural deflections are directly proportional to the dynamic pressure q and lift slope m; hence a desired α_I-distribution can be obtained by a simple linear superposition of these characteristic results using the fundamental parameter qm.

Authors choose for the preliminary cases the first three terms of a power series of the spanwise coordinate η. The application of the superposition method is illustrated in several tables.

The superposition method is not limited to finding the aerodynamic characteristics of swept wings. In reviewer's opinion, it is a simple and rapid method for solving all combined problems of aerodynamic and elastic forces.   N. Scholz, Germany

847. Diederich, F. W., Calculation of the aerodynamic loading of swept and unswept flexible wings of arbitrary stiffness, *NACA Rep.* 1000, 29 pp., 1950.

See AMR **3**, Rev. 561.

848. Tricomi, F. G., The airfoil equation for a double interval, *ZAMP,* **2**, 5, 402–406, Sept. 1951.

The note is in connection with author's paper "On the finite Hilbert transformation" [*Quart. J. Math.* **2**, 1951], in which the validity conditions of the known Söhngen solution of the airfoil equation are considerably extended. It is shown that the formula of Söhngen yields an explicit solution also when the integral of the airfoil equation is extended upon a *double interval*; this case occurs in the modern airplanes with sweptback wings. The solution thus obtained contains two arbitrary constants.

From author's summary

849. Riley, D. R., Wind-tunnel investigation and analysis of the effects of end plates on the aerodynamic characteristics of an unswept wing, *NACA TN* 2440, 55 pp., Aug. 1951.

Paper extends early subsonic aerodynamic investigations on effect of end plates on wings from both experimental and analytical viewpoints. Particular attention is given to improvements attainable in lift-drag ratio, as well as to the increase in lift-curve-slope and reduction in induced drag normally experienced.

Experimental studies were carried out on an NACA 64_1A412 airfoil of aspect ratio 4 at a Reynolds number of 10^6 and Mach number of 0.211. Plates of various shapes were tested, including rectangular, circular, and trapezoidal, as well as a series designed on the basis of the computed pressure field around the basic wing.

The theoretical analysis is based on the Prandtl lifting-line theory with the Jones edge-velocity correction as modified by Swanson and Crandall. An extended discussion comparing theoretical and experimental results is given with good agreement being obtained in general.   M. J. Thompson, USA

850. Gates, O. B., Jr., and Sternfield, L., Effect of an autopilot sensitive to yawing velocity on the lateral stability of a typical high-speed airplane, *NACA TN* 2470, 29 pp., Sept. 1951.

Using the conventional period-damping "flying-quality" criterion, report shows that an automatic pilot sensitive to angular velocity in yaw will give satisfactory lateral stability characteristics to an initially unsatisfactory fighter-class airplane. Study of effect of inclination of the gyro-axis indicates a range of acceptable variation. Time lag up to $1/10$ sec in the autopilot was found to have negligible effect on stability.

Paper is indicative of the wide gap now existing between level of technical methods for predicting the response of aircraft to arbitrary inputs and comparatively crude criteria expressing concensus of pilot opinion as to what constitutes satisfactory flying qualities.   Herbert K. Weiss, USA

851. McCullough, G. B., and Gault, D. E., Examples of three representative types of airfoil-section stall at low speed, *NACA TN* 2502, 52 pp., Sept. 1951.

Force, moment, pressure-distribution, and boundary-layer measurements are presented for a series of five airfoil sections. The stalling characteristics of these airfoil sections at low speeds are of three types: (1) Trailing-edge stall (preceded by movement of the turbulent separation point forward from the trailing edge with increasing angle of attack); (2) leading-edge stall (abrupt flow separation near the leading edge generally without subsequent reattachment); (3) thin-airfoil stall (preceded by flow separation at the leading edge with reattachment at a point which

moves progressively rearward with increasing angle of attack). The role of the boundary-layer flow and separation processes in relation to stalling as well as the sensitivity of the stall to factors which influence boundary-layer growth, such as Reynolds number, is discussed.　　　　　　　　　　From authors' summary

852. **Gadeberg, B. L., The effect of rate of change of angle of attack on the maximum lift coefficient of a pursuit airplane,** *NACA TN* 2525, 17 pp., Oct. 1951.

The effect of rate of change of angle of attack on the maximum lift coefficient of a pursuit airplane equipped with a low-drag-type wing has been investigated in stalls of varying abruptness over the Mach number range from 0.18 to 0.49 and Reynolds number range from 6.1 to 13.4 million.

The maximum lift coefficients were found to increase linearly with increasing rate of change of angle of attack per chord length of travel up to the maximum rate attained in the tests (0.66° per chord length of travel) in contradistinction to the results of the flight tests of two other airplanes.

The tests indicated that the Mach and Reynolds numbers effects were of sufficient importance to produce more than a twofold variation in the increment of $C_{L_{max}}$ due to a given rate of change of angle of attack.　　　From author's summary

853. **Bird, J. D., Jaquet, B. M., and Cowan, J. W., Effect of fuselage and tail surfaces on low-speed yawing characteristics of a swept-wing model as determined in curved-flow test section of Langley stability tunnel,** *NACA TN* 2483, 19 pp., Oct. 1951.

A wind-tunnel investigation was made to determine the influence of the fuselage and tail surfaces on the rotary derivatives in yawing flight of a transonic airplane configuration which had the wing and tail surfaces swept back 45°. The results of the determination of the rate of change of the yawing-moment coefficient with yawing velocity by two oscillation techniques agreed well with the determinations by the curved-flow procedure. The vertical tail was the main contributor to this derivative. The value for the complete model was essentially constant up to the angle of attack corresponding to maximum lift coefficient, and could be accurately calculated when proper account was taken of the end-plate effect of the horizontal tail on the vertical tail. The rate of change of rolling-moment coefficient with yawing velocity was mainly a contribution of the wing. This derivative increases approximately linearly with angle of attack to the angle of attack where the curves of lift and pitching-moment coefficient plotted against angle of attack develop non-linearities.　　　From authors' summary

854. **Jones, W. P., A note on the use of time series in the analysis of flight test series,** *Aero. Res. Counc. Lond. curr. Pap.* 46, 24 pp., Jan. 1950, published 1951.

Paper deals with response of linear systems to arbitrary inputs. Author develops the method of Tustin [*J. Instn. elect. Engrs.* 94 (IIA), 1, 1947] in matrix notation, and discusses possibility of applying it to analysis of flight-test data on stability and control. He shows how the response of an airplane to a unit impulse type of input in any degree of freedom can be found, by this method, from flight records of the response to an arbitrary forcing function in that degree of freedom.

Although some trial calculations have been made, the method has not yet been put to the test. Author expresses some doubts (shared by the reviewer) that the accuracy will be sufficient.

Essence of the method is that time-dependent functions [e.g. input $e(t)$ and response $r(t)$] are given by column matrixes of the values at uniform time intervals δ, i.e., $\{e\}$ and $\{r\}$. When the response to a triangular input function (Δ impulse) is given by $\{d\}$, the numerical form of Duhamels' integral for the response is the matrix equation $\{r\} = A(d) \{e\} \ldots$ [1] in which $A(d)$ is a simple triangular matrix derived from $\{d\}$.

Author inverts this equation to obtain $\{d\} = [A(e)]^{-1} \{r\} \ldots$ [2]. It is equation [2] which enables the response to unit impulse $\{d\}$ to be found from the measured response $\{r\}$ to an arbitrary input $\{e\}$.　　　　　Bernard Etkin, Canada

Aeroelasticity (Flutter, Divergence, etc.)

(See also Rev. 847)

855. **van de Vooren, A. I., and Hofsommer, D. J., Ternary wing bending-aileron-tab flutter,** *Nat. LuchtLab. Amsterdam Rap.* F.86, 29 pp., 2 tables, 22 figs., May 1951.

Report consists of theoretical study limited to the case of zero frequency tab and subsonic flow with several other restrictions regarding the mass, elastic, and geometric parameters. Results show (a) stability of each two degree-of-freedom subsystem does not guaranteee stability of complete system; (b) measures favorable for flutter prevention include small tab moment of inertia, large aileron moment of inertia, and avoiding mass underbalance or overbalance of tab.　　　　Benjamin Smilg, USA

856. **Greidanus, J. H., and van de Vooren, A. I., Station functions in flutter analysis,** *J. aero. Sci.* 17, 3, 178–179, Mar. 1950.

Advantages of the station-functions method of Rauscher are questioned by authors. Points of controversy are discussed in Rauscher's reply.　　　　　W. T. Thomson, USA

857. **Söhngen, H., Schwarz, L., and Dietze, F., Three papers from conference on "wing and tail-surface oscillations"— March 6–8, 1941, Munich,** *NACA TM* 1306, 47 pp., Aug. 1951.

Translation from *Lilienthal-Gesellschaft für Luftfahrtforschung*, Ber. 135.

858. **Baird, E. F., and Kelley, H. J., Formulation of the flutter problem for solution on an electronic analog computer,** *J. aero. Sci.* 17, 3, 189–190, Mar. 1950.

Authors write flutter equations of motion in real plane and ready same for solution on analogue computers. Procedure is well known.　　　　　Y. Luke, USA

859. **Laasonen, P., On the theory of flutter and an iterative method of calculating the critical speed of a wing,** *Roy. Inst. Technol., Div. Aerodyn., Stockholm, KTH-Aero TN* 11, 12 pp., 1950.

Report contains general introduction to flutter problem. The eigenvalue problem for an oscillating bending and twisting wing is established in the form of differential equations. An iterative method is presented for solving the smallest eigenvalue (frequency) with corresponding flexural and torsional deformations. Author states that he has proved the convergency of the method for special cases only. Reviewer remarks that other, definitely converging, iteration procedures are available, allowing determination of the eigenvalues in order of increasing modulus [e.g., Wieland, *ZWB U.M.* 3138, 1944; or Greidanus and van de Vooren, AMR 4, Rev. 1355].

A. I. van de Vooren, Holland

860. **Ghaswala, S. K., Aerodynamic aspects of civil engineering,** *Civ. Engng. Lond.* 45, 531, 532, 533; 586–589, 646–649, 725–727; Sept., Oct., Nov. 1950.

Articles review aerodynamics for civil engineers. Oscillations of suspension bridges, wind loads on structures and buildings, use of

wind tunnels for obtaining such loads, and pressure-supported roofs are discussed. Paul A. Libby, USA

Propellers, Fans, Turbines, Pumps, etc.

(See also Revs. 587, 588, 615, 877, 879)

861. Lerbs, H. W., On the effects of scale and roughness on free running propellers, *J. Amer. Soc. nav. Engrs.* **63**, 1, 58–94, Feb. 1951.

Author develops methods for determining the full scale characteristics (k_T vs. J and k_Q vs. J) of ship propellers from the characteristics of model propellers. The paper is in two parts; the first considers the case when only the drag coefficient of the airfoil profile depends on Reynolds number and roughness. Second part includes variation of both lift and drag with Reynolds number and roughness.

Since the methods take the over-all characteristics k_T and k_Q as starting point and then revert to the lift-drag polar of an airfoil section as basis for extrapolation of the propeller full-scale characteristics, some intermediate assumptions are necessary. The k_T and k_Q vs. J curves of a propeller are the integrated results of the contributions of every part of the propeller. To pass from the integrals to a reasonable evaluation of the lift-drag polar of any or all of the blade sections requires assumption of the soundness of current design procedures for ship propellers.

Author makes a strong case for the possibilities of obtaining reasonable results. Dino Morelli, USA

862. Delano, J. B., Investigation of the NACA 4-(5)(08)-03 and NACA 4-(10)(08)-03 two-blade propellers at forward Mach numbers to 0.725 to determine the effects of camber and compressibility on performance, *NACA Rep.* 1012, 31 pp., 1951.

Tests of two-blade propellers were made in an 8-ft high-speed tunnel through a range of blade angle from 20 deg to 60 deg for forward Mach numbers from 0.165 to 0.70 to determine the effect of camber and compressibility on propeller characteristics.

Blades of high-design camber were more efficient than blades of low-design camber operation at high power loadings. The medium-camber propeller gave peak efficiencies 2 to 5% higher than the low-camber propeller and 3 to 12% higher than the high-camber propeller. The range of power disk-loading coefficient over which high efficiencies could be obtained was greatly reduced at high speeds. R. C. Binder, USA

863. Flügel, G., Steam turbines during large changes in operation (in German), *ZVDI* **93**, 22, 721–728, Aug. 1951.

For steam turbines, knowledge of the moment, the output, and the efficiency at working conditions very different from the normal is often desirable. In such a case, especially if the speed varies very much from the normal, a severe impact will be caused at the entrance of the guide and moving blades. Author investigates theoretically the behavior of single stages and groups of stages at steam velocities under and over the velocity of sound, with respect to the loss due to impact at the entrance and the contraction of the outlet cross section. In comparison with the empirical rule of Stodola, the paper shows an essential progress. Pavel Kohn, Czechoslovakia

864. Ellerbrock, H. H., Jr., Some NACA investigations of heat transfer of cooled gas-turbine blades, Gener. discuss. heat transfer Lond. Conf., Sept. 11–13, 1951, Sect. V. London, Instn. mech. Engrs.; New York, Amer. Soc. mech. Engrs., 11 pp.

Information on heat-transfer characteristics and temperature distribution of cooled gas-turbine blades with large gas-to-blade-wall temperature ratios is given. Regarding gas-to-blade heat transfer, relatively good correlation of experimental results was obtained by basing all gas properties on blade-wall temperature, while Reynolds numbers were based on average gas velocity around blade. A theoretical calculation of heat-transfer coefficients is given. Sufficient agreement with measurements is reached, enabling calculation of blade-wall temperature distributions. Experimental results and calculation methods are given for blade-to-coolant heat-transfer coefficients for air- and water-cooled blades. Measurements and theoretical methods for transpiration cooling with porous blade materials are listed, whereby the proposed theory does not accurately agree with the experimental results. E. Haenni, Switzerland

865. Ellis, G. O., and Stanitz, J. D., Two-dimensional compressible flow in centrifugal compressor with logarithmic-spiral blades, *NACA TN* 2255, 46 pp., Jan. 1951.

Authors apply methods previously developed by the second author to investigate flow in a compressor where the center line of the flow passages lies in a right circular cone, the axis of which is coincident with the axis of the compressor.

The quantitative results, obtained by relaxation-method techniques, indicate that a reduction in eddy flow may be attained with backward or forward curved blades as compared to radial blades. The suggestion of improved adiabatic efficiency follows immediately. Indicated constancy of static pressure ratio between straight and curved blades suggest that the diffuser problem is less critical with backward curved blades.

Not intended for establishing design criteria, the computed results clearly indicate the desirability of experimental investigations upon which design criteria may be based.

J. F. Manildi, USA

866. Quenzer, H., and Schwarz, G., Aerodynamic calculation methods for high power axial compressors (in German), *Schweiz. Bauztg.* **69**, 31, 33; 432–435, 462–465; Aug. 1951.

A survey of practical compressor design, including the degree of reaction, its radial variation, and optimum value. Simple radial equilibrium is considered. It is shown that under limitation of Mach numbers the delivery head is also limited. Defining M_u as the ratio of tip speed to sound velocity, if there are no inlet guide vanes, the limitation in delivery head will arise in the rotor for $M_u < 0.7$, and in the stator for $M_u > 0.7$. The limits may be raised by use of inlet guide vanes of radially constant circulation and further increased by favorable nonconstant radial distribution of the circulation. Besides the fixed inlet guide vane, the possibilities of inducers are mentioned. These include inducers with reduced tip speed by decreased radius as well as by reduced turning speed, and inducers with additional stators. The reduced turning speed and relatively low load permit light weight of the gearing. For the selection of profiles, use of NACA data is mentioned as preferable to mere choice of profiles having circular arc camber lines.

Besides exaggeration of the leading- and trailing-edge angles as based on the theory of thin profile cascades, there are considered by approximate methods the influence of the thickness ratio and—in case of nonconstant circulation inlet guide vanes—the induced disturbances. The influence of compressibility is also considered. Friedrich Weinig, USA

867. Reissner, H. J., and Meyerhoff, L., Analysis of an axial compressor stage with infinitesimal and finite blade spacing, *NACA TN* 2493, 32 pp., Oct. 1951.

Paper contains an extension of the work reported in references 1 and 2, which treat the flow through an annulus containing an infinite number of vanes, to the more physically realistic case of

a finite number of vanes with arbitrary spacing. This extension eliminates the axial symmetry which existed in the problems of references 1 and 2, and the equations are, therefore, complicated by the necessary inclusion of the derivatives with respect to ϕ, (the angular cylindrical coordinate).

In order to solve this set of equations, the solutions for the infinitesimal spacing are used as a zero-order approximation. The solutions of the equations are then expressed as follows:

$$u = u_0 + \psi u_1(r, z) + \psi^2 u_2(r, z) + \dots$$
$$v = v_0 + \psi v_1(r, z) + \psi^2 v_2(r, z) + \dots$$
$$w = w_0 + \psi w_1(r, z) + \psi^2 w_2(r, z) + \dots$$
$$p = p_0 + \psi p_1(r, z) + \psi^2 p_2(r, z) + \dots$$

where u, v, w are velocity components along cylindrical coordinate directions (r, ϕ, z), P is pressure, u_0, v_0, w_0, p_0 are the zero-order solutions, and ψ is an angular parameter. By substituting these formal solutions into the equations, and equating coefficients of like powers of ψ, it is possible to evaluate the variables u_1, v_1, w_1, and p_1 and also u_2, v_2, w_2, p_2, etc. In the process of solution for these variables, however, there also appear terms such as $\partial \psi / \partial r$, $\partial \psi / \partial \phi$, $\partial \psi / \partial z$, which must be assumed in a manner which is appropriate for the satisfaction of the boundary conditions on the blades.   Morton Alperin, USA

868. Oguey, P., Mamin, M., and Baatard, F., Theoretical and experimental study of the dispersion of the jet in Pelton turbines (in French), *Bull. tech. Suisse Rom.* 77, 4, 5; 37–47, 53–64; Feb., Mar. 1951.

Dispersion of the jet in 56- and 45-mm diam Pelton nozzles, with and without needle, is examined by measuring the velocity field downstream of the nozzle with Prandtl tubes. Characteristic coefficients to judge the jet quality are determined. The model laws of Pelton jets are especially treated, and authors come to the important conclusion that the dispersion and velocity field of water jets are similar whenever geometrical similarity of the nozzles and equal Reynolds number of the jets exist.

Moreover, the effect on the quality of the jet resulting from nozzle shape, guide cross, and diameter of jet, as well as the head, is discussed. Photos of formation of water jet, given in this paper, are worthy of note. This experimental and theoretical study will be a valuable aid to the hydraulics engineer.

E. Mühlemann, Switzerland

869. Moyes, S. J., and Pennington, W. A., The influence of size on the performance of turbojet engines, 3rd Anglo-Amer. aero. Conf., Brighton, Sept. 4–7, 1951. Roy. aero. soc., 545–562.

In considering the thermodynamic cycle on which the turbojet engine is based, a simple analysis will show that, for a given flight condition—turbo inlet temperature; compressor ratio; compressor-, turbine-, combustion- and nozzle-efficiency—the thrust per unit of mass flow is constant. If, under these conditions, a series of geometrically similar engines are considered having the same peripheral speed, the thrust and the engine weight are proportional to the square and the cube of a characteristic dimension, respectively. Hence, scaling down would offset the weight per unit of thrust, leaving the specific fuel consumption unchanged. In fact, however, scaling up or down upsets or offsets the compressor and turbine efficiency due to the variation of Reynolds number. Variation of height causes a similar effect. Furthermore, in scaling down, e.g., geometrical similarity can be maintained only for certain parts of the engine and only over a certain range.

Starting with a chosen basic engine, specific engine weight, fuel economy and relative thrust per unit of frontal area are given as a function of sea level design air mass flow for some flight conditions, taking into account the effect of variation of Reynolds number and the effect of manufacturing limitations when scaling up or down. Specific fuel consumption increases while scaling down; the specific engine weight curve shows a minimum.

J. G. Slotboom, Holland

870. Suter, A. T., and Zipkin, M. A., Method of matching components and predicting performance of a turbine-propeller engine, *NACA TN* 2450, 75 pp., Sept. 1951.

Equations are derived for the equilibrium operation of a turbine-propeller engine in terms of parameters used in turbine- and compressor-performance maps. Using these analytical expressions, geometric, thermodynamic, and aerodynamic relations among compressor, turbine, and exhaust nozzle are calculated.

Once a particular compressor-performance map is chosen, a simplified systematic method is developed to show the necessary compromise in the selection of the turbine to be mated with the compressor. Results of the component matching are used to obtain the predicted power per unit size of the engine. The method of performance analysis permits the prediction of the over-all engine performance and some of the limitations such as compressor surge and maximum turbine-pressure ratio. The matching relations may be used to predict engine performance over a range of operating conditions after the physical relations and the components are known.

Considering the complexity of the problem, due to the many variables involved, authors' presentation is brief. A helpful example of the matching method and the performance analysis is presented for an axial-flow compressor and a single-stage turbine coupled to a constant efficiency propeller.

H. E. Sheets, USA

871. Wu, C.-H., and Brown, C. A., Method of analysis for compressible flow past arbitrary turbomachine blades on general surface of revolution, *NACA TN* 2407, 42 pp., July 1951.

Paper is one of a series published recently by the senior author, and deals with the direct problem (i.e., blade shape given) in subsonic flow. Purpose is to obtain the shape of the mean streamline and specific mass flow variation along this line, for "through-flow" analysis [AMR 4, Rev. 3297] of turbomachines with thick blades. To use method of this paper, stream surfaces of revolution corresponding to the given blade row must either be known from an approximate through-flow analysis or else be assumed. Equations of motion are then reduced to one second-order nonlinear stream-function equation, to be solved numerically by an iteration procedure: the nonlinear terms (containing density and its derivatives) are held constant during each cycle. Resulting approximate linear equation is solved by finite difference methods. Fourth-degree Lagrangian polynomials are suggested as most suitable for typical case, with use of differentiation coefficients given in a previous paper. Three methods for solving difference equations are discussed and formulas are given for estimating residual error.

Method was applied to cascade of typical high solidity highly cambered turbine blades on a cylindrical surface for two cases, incompressible flow and inlet Mach number 0.42. Relaxation solution for compressible flow case shows good agreement with the experimental velocity distribution. (No details are given on the experiments used for comparison.) Velocity distributions across the channel, mean stream-, channel-, and camber lines and specific mass flow variation on mean streamline are given for both theoretical solutions. These results should (1) serve as a basis for devising a simple approximate method for two-dimensional cascades, and (2) give useful information for "through-

flow" analysis of turbines, i.e., shape of mean streamline and correction factor for finite blade thickness.

In reviewer's opinion, the general approach to the problem involves a crucial assumption which is not emphasized: The stream surfaces of revolution obtained by "through-flow" (i.e., infinite number of blades) analysis are presumed to be undistorted in change to finite number of blades; only changes of flow distribution *within* these surfaces are considered. The validity of this assumption for other than very high solidity and the importance of velocity perturbations normal to the original surfaces of revolution require further clarification. Also, some information on convergence of the iteration procedure near a sonic line would be desirable. On the whole, the method should be of great value for certain design problems and for further theoretical work.

Rolf D. Buhler, USA

872. Rashed, M. I. I., Pressure oscillations in a centrifugal pump and analytical determination of the pump characteristics (in German), Thesis, Inst. Hydraul. hydraul. Masch., *ETH, Zürich, Mitt.* **3**, 55 pp., 48 pp. of figs., 1951.

Paper analyzes the effect of blade number, speed, and capacity on pressure and velocity oscillations. A mathematical analysis is made for a single case using equations based on Spannhake's fundamental theories. This analysis treats the two-dimensional case of an impeller with parallel walls and radial blades. A graphical method is used for the general case of a centrifugal pump. Both the mathematical analysis and the graphical method are used to analyze periodic pressure variations at various locations, particularly at the exit of a centrifugal impeller.

Tests are reported to verify the analytical investigations. The data indicate that the maximum pressure oscillations occur when the pump is operated at about one-half capacity. The largest oscillations are accompanied by maximum noise and it would be desirable to reduce these oscillations to a minimum. The analysis also includes the effects of guide vanes, diffuser, and scroll on the pressure oscillations, and it is shown that a larger distance between guide vanes and impeller acts favorably to reduce pressure oscillations. The analysis indicates also that the number of impeller blades and impeller speed should be so selected that no amplification of the pressure waves is induced by the scroll structure.

The flow in the inlet pipe is also investigated. These studies show that at a partial load the velocity, the pressure, and the total energy of the fluid near the outer wall are larger than in the center. A back flow of the fluid in the suction pipe is observed. This results in a prerotation of the flow. The prerotation makes it difficult to predetermine pump characteristics. In addition, it causes some reduction in efficiency at partial load due to the back flow of the fluid in the suction pipe. The prerotation, on the other hand, reduces the problems of cavitation for partial load.

Paper is quite a remarkable study. It presents a rather complete analysis of the performance of turbomachinery with finite number of blades based on periodic pressure and velocity variations at the impeller exit. While some data of pressure variations and prerotation have been reported previously in the literature, this paper presents a complete study with the latest information on pressure variations in turbine machinery.

H. E. Sheets, USA

873. Railly, J. W., The flow of an incompressible fluid through an axial turbo-machine with any number of rows, *Aero. Quart.* **3**, part 2, 133–144, Sept. 1951.

An exact mathematical treatment of the general problem being impossible at present time, following simplifications are made: The nonviscous, axisymmetric flow takes place between cylin-

drical walls; the blade rows are reduced to infinitely thin lifting disks.

In the case of an isolated row, a simplified expression for the radial velocity component is assumed (Traupel, Marble):

$$u = [\exp(\pm k_i |z - (i - 1)\beta|)] f_i(r); \quad z \lessgtr (i - 1)\beta$$

Integration of the continuity equation yields thus the axial velocity component $w(z, r)$ and especially the well-known result (Ruden): The axial velocity at the row is midway between the velocity far upstream and far downstream. Constants k_i and $f_i(r)$ are calculated by reference to conditions of flow at the row, neglecting hereby actual radial displacements of the streamlines. Solution is complete, provided that axial velocity distribution at the row is given.

Problem of the multistage machine is solved under assumption that radial velocity field results from superposition of fields due to each of the rows acting individually. No justification of this procedure is given. In fact, this implies a linearization of governing differential equation for the stream function [AMR 4, Rev. 3031]. Since neither axial nor radial velocities are initially known, calculation results in successive approximations.

Problem of the periodic machine with infinitely many rows is treated along similar lines. Two results are reported: (a) Axial velocity is the same in plane of rotor and stator; (b) work done is independent of radius.

A single compressor stage with untwisted blades and axial upstream velocity is worked out as an example. Three approximations for axial velocities are calculated, showing good convergence. Finally, this flow pattern is compared with the ultimate periodic one for same blading.

Reviewer notes that a more systematic treatment of the whole question involving the same initial simplifications has been given by Siestrunck and Fabri (op. cit.), who offer further complete and rigorous discussion of simplifying assumptions. Thus, one will be informed that, within limits of theory, statement (b) has general validity, whereas (a) implies linearization assumptions.

Pierre Schwaar, Switzerland

874. Rütschi, K., Reassessment of the efficiency of pumps and turbines (in German), *Schweiz. Bauztg.* **69**, 38, 525–527. Sept. 1951.

The conversion of the efficiency from model test to prototype by a simple formula is very important in practice. Systematic tests performed on centrifugal pumps during recent years have produced new knowledge. It became evident that the relative roughness, which depends essentially on the size of the machine, is of great influence, while up to now only the effect of Reynolds number was assumed. Author quotes a new conversion formula, proposed by Prof. Pfleiderer, which takes into consideration the dependency on the size of the pump, as well as on Reynolds number. Further tests on hydraulic turbines are recommended and it is hoped that the experts will accept the new reassessment formula.

Pavel Kohn, Czechoslovakia

875. Kaye, J., and Wadleigh, K. R., A new method of calculation of reheat factors for turbines and compressors, Ann. Meeting, Amer. Soc. mech. Engrs., Atlantic City, 1951. Paper no. 51—A-2, 6 pp. = *J. appl. Mech.* **18**, 4, 387–392, Dec. 1951.

For rapid calculation of reheat factors for adiabatic turbines and compressors, a method is given based on the concept of the adiabatic efficiency η_∞ of an infinitesimal expansion or compression step and on the relative pressure function p_r defined by

$$\ln p_r = R^{-1} T_0 \int^T c_p dT / T$$

The method is limited to fluids for which (a) the known equation

of state $pv = RT$ is valid, and (b) the specific heat is a function of temperature only.

Charts of reheat factors for turbines and compressors with several gases are produced, using "Gas tables" by J. H. Keenan and J. Kaye, in which p, and the enthalpy are tabulated.

J. G. Slotboom, Holland

Flow and Flight Test Techniques

(See also Revs. 619, 758, 761, 803, 849, 853, 898, 907)

876. **Berndt, S. B., Wind tunnel interference due to lift for** delta wings of small aspect ratio, *Roy. Inst. Technol. Div. Aero., Stockholm, KTH-Aero TN* 19, 11 pp., 1950.

Using the slender-wing theory of Jones, tunnel-wall corrections for incompressible flow are given for the angle of incidence, resistance, and pitching moment. In connection with the small aspect ratio, the wing and the wake are replaced by a dipole distribution on the long axis. Interference factors are calculated for triangular wings in closed rectangular test sections. The obtained corrections differ little from those obtained by litting-line theory. H. G. Loos, Holland

877. **Bundy, F. P., Strong, H. M., and Gregg, A. B., Meas**urement of velocity and pressure of gases in rocket flames by spectroscopic methods, *J. appl. Phys.* 22, 8, 1069-1077, Aug. 1951.

Paper describes spectroscopic measurements of gas-flow velocity and static pressure in the gas issuing from an oxygen-alcohol rocket. The method depends upon the measurement of the shift in wave length of sodium or lithium radiation from the flame; the Doppler shift due to pressure by viewing the rocket jet at different angles. The absolute accuracy obtained is not very impressive (velocity to $\pm 1.5 \times 10^4$ cm/sec, pressure to ± 0.3 atm), but the experiments have shown that the use of these optical techniques is quite practical. Their inaccuracy is explainable in part by the fact that they give an average value for pressure and temperature over an optical path through the rocket jet where the local conditions vary quite widely. R. Smelt, USA

878. **Krause, L. N., Effects of pressure-rake design** parameters on static-pressure measurement for rakes used in subsonic free jets, *NACA TN* 2520, 20 pp., Oct. 1951.

Results obtained in a subsonic-free-jet investigation conducted to determine the effect of pressure-rake design parameters on static-pressure measurement over a range of Mach numbers from 0.3 to 0.95:

The effect of variation in the distance from the leading edge of the static-pressure tube to the static orifices was small compared with the effect of the distance from the static orifices to the supporting strut. The effect of variation in the ratio of support diameter to jet diameter became pronounced at high Mach numbers. Proximity effects of adjacent tubes near static-pressure tubes may be alleviated by proper orientation of the leading edge of the adjacent tube in relation to the static orifices.

From author's summary

879. **Lalos, G. T., A sonic-flow pyrometer for measuring** gas temperatures, *J. Res. nat. Bur. Stands.* 47, 3, 179-190, Sept. 1951.

Advantages of a sonic-type suction pyrometer over conventional instruments are discussed. Shielding a thermocouple from radiation and locating it at the throat of a nozzle through which the hot gas is made to flow with sonic velocity allows accurate measurement of total temperature. A correction factor depending on γ for the gas and the recovery factor of the pyrometer can be applied to the observed temperature. Calibrating at room temperature and applying a constant correction factor gives an accuracy of measurement of about 1% up to 1500 F. The theory of operation, method of calibration, and details of construction are given.

J. C. Wisdom, Australia

880. **Stever, H. G., Condensation of air in hypersonic wind** tunnels, Gener. discuss. heat transfer, Lond. Conf., Sept. 11-13, 1951, Sect. V. London, Instn. Mech. Engrs.; New York, Amer. Soc. mech. Engrs., 4 pp.

In wind tunnels achieving very high Mach numbers, air is expanded to such an extent that the temperature in the test section may fall below its liquefaction point. This paper is an experimental and theoretical attack on the problems of nucleation which then arise. Its theoretical contribution extends the Becker and Doring theory to the case when the nucleus is very small, i.e., contains a relatively small number of molecules, as is the case in practice in wind-tunnel application. Paper calculates an approximate surface tension-drop radius relation, and uses this to derive oxygen and nitrogen condensation rates as a function of temperature and pressure.

Experiments on a small hypersonic wind tunnel to examine liquefaction at high Mach numbers are also described; condensation is detected by light-scattering from the condensed droplets in the test section. Conditions at the commencement of liquefaction are compared with the theory.

R. Smelt, USA

881. **Dubois, G., Kling, R., and Vieilledent, E., Study of** transonic or supersonic flow fields by means of ultrasonic waves (in French), *C. R. Acad. Sci. Paris* 233, 2, 129-131, July 1951.

Paper describes use of ultrasonic waves to measure temperature of a high-velocity gas stream by optical measurement of wave length and calculation of local speed of sound. Method was first developed to measure stream velocities, but is limited because of diffraction phenomena at edges of wave band. In experimental work, strong absorption of high-frequency ultrasonics was encountered; measurements of flow in nozzle throat were made with low frequencies (22,000 cps). Interaction of shock waves with ultrasonics is described as a possible cause of absorption of high frequencies. A. Shaffer, USA

882. **Reichardt, H., On the recording of turbulent longi**tudinal and transverse fluctuations, *NACA TM* 1313, 10 pp., Aug. 1951.

Translation from *ZAMM* 18, 6, 1938.

883. **Malavard, L., On a new technique in experimental** solutions by rheoelectric analogies (in French), *Rech. aéro.* no. 20, 61-68, Mar.-Apr. 1951.

Boundary-value problems involving Laplace or associated equations can be interpreted as problems of current distribution in conductors, and the latter solved experimentally. Author and others have demonstrated practical value of electric analogy in numerous applications (Bibliography). Paper describes use of paper conductor (commercially available Télédeltos) in place of usual electrolytic basin for study of electric problem.

Brief review of application to problems in spanwise lift distribution, mapping, temperature distribution, and velocity distribution on multislotted airfoil indicates essential advantages of simplicity, flexibility, speed, and low cost of model fabrication with this new procedure. Accuracy is only slightly inferior to basin in most cases. Simultaneous use of paper conductor and

electrolytic basin can extend field of practical application of electric analogy.   J. S. Isenberg, USA

884. **Rasmussen, R. E. H., The flow of gases in narrow channels**, *NACA TM* 1301, 46 pp., Aug. 1951.

Translation from *Ann. Phys., Leipzig,* 29, 8, 1937.

885. **Redshaw, S. C., and Palmer, P. J., The construction and testing of a xylonite model of a delta aircraft**, *Aero. Quart.* 3, part 2, 83-127, Sept. 1951.

In order to provide information to assist with the unorthodox design of a delta aircraft, an accurate scale model in Xylonite (celluloid) was designed and constructed. The reasons for the selection of the particular scale factors so that the model could be used for stiffness, strain distribution, resonance, and flutter tests are discussed. A description is given of the special test instruments which were developed, together with an account of the control tests and the tests to which the model was subjected. Finally, a comparison is made with the actual tests which were made on the full-scale aircraft at a later date. The full-scale and model results were found to be in good agreement.

Alfred J. Eggers, Jr., USA

886. **Berndt, S. B., Three component measurement and flow investigation of plane delta wings at low speeds and zero yaw**, *Roy. Inst. Technol. Div. Aero., Stockholm, KTH-Aero TN* 4, 19 pp., 1949.

Three component measurements and stream flow tests at low speeds have been made at zero yaw on 6 plane delta wings, 3 of which are triangular and 3 tapered with taper ratio 0.3. The aspect ratio of the wings is between 0.5 and 2.5. At small angles of attack, the experimental values of lift and pitching moment are in agreement with calculations based on linear wing theory. At greater angles of attack, a nonlinearity is observed, which rapidly increases with decreasing aspect ratio, and which may be due to a backwards movement of the lift distribution. The separation of the flow begins at the wing tips, and spreads inward with increasing angle of attack so slowly that even at an angle of attack of 40 deg, it does not cover the whole wing.

From author's summary

887. **Harper, J. J., Investigation at low speed of 45° and 60° sweptback, tapered, low-drag wings equipped with various types of full-span, trailing-edge flaps**, *NACA TN* 2468, 53 pp., Oct. 1951.

Although emphasis was placed on attempts to obtain higher maximum lift coefficients, the results show no appreciable gain in $C_{L_{max}}$ for either the split flap or the special flaps on the 60° wing. These same flaps did produce an increment in $C_{L_{max}}$ on the 45° model, however. The slotted flap produced by far the largest increase in lift on both models.

The effect of the flaps on drag and pitching moment is the same as that indicated by other test data available. The slotted flap caused the largest increase in pitching moment for a given deflection angle. All flap configurations increased the stability of both wings.   From author's summary

Thermodynamics

(See also Revs. 784, 785, 819, 877, 880, 884, 896, 897)

888. **Hoge, H. J., and Arnold, R. D., Vapor pressures of hydrogen, deuterium, and hydrogen deuteride and dew-point pressures of their mixtures**, *J. Res. nat. Bur. Stands.* 47, 2, 63-74, Aug. 1951.

The vapor pressures of H_2, HD, and D_2 have been measured from near their triple points to their critical points. H_2 and D_2 samples were catalyzed to ortho-para equilibrium at 20.4 K. Tables suitable for interpolation have been prepared to represent the results both in centimeter-gram-second and in engineering units. Measurements of dew-point pressures of several binary mixtures have been made at several pressures below atmospheric. Observed pressures were about 3% above those predicted by the law of ideal solutions.

From authors' summary by Y. S. Touloukian, USA

889. **Wild, E., On Boltzmann's equation in the kinetic theory of gases**, *Proc. Camb. phil. Soc.* 47, part 3, 602-609, July 1951.

Boltzmann's differentio-integral equation is integrated to yield a pure integral equation. Using iteration process, different from Hilbert and Carleman, author finds nonstationary solution to "initial value problem" in unbounded space, when mean collision frequency is assumed a known function of space, time, and the velocity of a colliding molecule. Under certain restrictions on initial distribution, it is shown that this iteration converges in a finite time interval. Application of method is made to gas in which the molecular collision cross section varies inversely with the relative velocity of colliding molecules for initial distributions independent of space.   Paul F. Byrd, USA

890. **Klobe, G., The adiabatic exponent of dissociating products of combustion at adiabatic-isentropic expansion** (in German), *ZAMP* 2, 5, 394-402, Sept. 1951.

Paper attempts to show how the degree of dissociation and the exponent of the adiabatic p, T and p,v curves of products of combustion of hydrocarbon fuels at high temperatures can be approximately calculated from empirical and semi-empirical relations. In developing these relations, author disregards entirely the correct thermodynamical procedure of calculating these quantities from the specific heats of the constituents and the latent heat of the reactions involved (as well as from their known variation with temperature and pressure) as presented nearly 30 years ago by W. Schüle in chapter IV of vol. II, 4th ed., of his textbook, "Technische Thermodynamik."

Eric F. Lype, USA

891. **Haase, R., Contribution to the thermodynamics of irreversible processes. II** (in German), *Z. Naturforsch.* 6a, 10, 522-540, Oct. 1951.

It is repeatedly asserted that stationary nonequilibrium states are specified by a minimum of entropy production. In this paper the validity of this assertion is investigated for a number of different irreversible processes; in particular, chemical reactions and transport phenomena. It is shown that under appropriate conditions—formulated explicitly—the rate of entropy production in a stationary state is smaller than in any neighboring accessible state. This minimum principle is not identical with a similar principle formulated by Onsager. The argument depends partially on the validity of linear relations between "forces" and "flows." The conclusion is nevertheless empirically known to apply to stationary explosion zones where linear relations are not valid. Author points out that the minimum principle does not guarantee stability of stationary states under consideration.   R. Eisenschitz, England

892. **Keenan, J. H., Availability and irreversibility in thermodynamics**, *Brit. J. appl. Phys.* 2, 7, 183-192, July 1951.

Author points out that the concept of an idealized process with which an actual process may be compared is common to all

branches of thermodynamics, but that the idealized processes usually selected lack generality. His purpose here is to develop more fully than has hitherto been done the general approach of J. W. Gibbs.

He shows that if a system exposed only to an infinite atmosphere changes from state 1 to state 2, the *useful* work W_u (work in excess of that done against the atmosphere) which could be obtained is equal to or less than $\Phi_1 - \Phi_2$, where $\Phi = E + p_0 V - T_0 S$, E denoting the energy of the system, V its volume, S its entropy, p_0 the pressure of the atmosphere, and T_0 its temperature. He points out that for any given state 1 the maximum possible value of W_u is $\Phi_1 - \Phi_{min}$, where Φ_{min} is the minimum value of Φ for all possible states (and corresponds to the most stable state). He proposes to call this $(W_u)_{max}$ the *availability* of the system at the given state.

A system exposed both to an infinite atmosphere and to a heat reservoir is considered next, and an expression for $(W_u)_{max}$ is derived. A closed control surface in a field of fluid flow exchanging heat only with the atmosphere is then treated, and expressions are developed for the maximum possible *shaft work* which can be delivered when an element of mass enters. Modified forms for steady flow are presented.

A quantitative definition of irreversibility is proposed. For a system exposed only to an infinite atmosphere it is proposed to define the irreversibility of any process between states 1 and 2 as $(W_u)_{max} - W_u = (\Phi_1 - \Phi_2) - W_u$. The irreversibility involved if the system receives heat from a reservoir and the irreversibility accompanying flow across a control surface are also considered.

Author explains that various coefficients of performance can be devised which increase in magnitude with decrease in irreversibility. For example, a coefficient appropriate for processes intended to produce useful work may be defined as $W_u/(W_u)_{max}$.

Finally, to show the generality of the method, author analyzes a variety of thermodynamic phenomena.

H. J. Stoever, USA

893. **Sprenger, H., Observations on vortex tubes** (in German), *Z.A.M.P.* **7**, 4, 293–300, July 1951.

By expanding a gas tangentially in a tube, cold gas can be withdrawn from one end and heated gas from the other. The mechanism of this process is not too well understood. The paper gives experimental results obtained on such a "vortex tube": The temperature of the gases as a function of the amounts withdrawn from the two ends; the heat balance; the frequency of the vibrations set up in the tube due to the process; and the temperature distribution along the walls.   Lothar Meyer, USA

894. **Lichty, L. C., Internal combustion engines, 6th ed.**, New York, McGraw-Hill Book Co., Inc., 1951, 598 pp. $7.

For years Lichty's book on internal-combustion engines has been an outstanding text as well as reference work for the practicing engineer. The new (sixth) edition follows the previously established pattern which includes thermodynamics, structure, and performance. A goodly number of examples show current practice, yet the book is not burdened with excerpts from catalogues. Since the last edition (1939) there have been many new developments in automotive engines. These have been treated and correlated with basic theory. Probably the most important addition is the inclusion of gas turbines and rocket engines.

The pertinent cycle analyses as well as an extensive treatment of fuels other than gasoline and fuel oil are included. Numerous references give material for further study.

The chapter on abnormal combustion, formerly called "Detonation and knock testing," has been renamed "Combustion knock and knock rating" in accordance with present practice, and the contents of the chapter have been considerably refined. The effect of design and operating variables on octane requirements is discussed at length, and the concept of knock-limited performance is given its full significance.

A short section on lubricating oil additives has been added, although no references are given, which is disappointing in such an important subject. A comprehensive section on cylinder wear is added, including numerous references which will ultimately expose the reader to additional data on additives.

A comparison of high-speed engine indicators is given in the section on performance.

In spite of all these additions, the number of pages has been kept the same, which shows that the existing material has been carefully reviewed.

In reviewer's opinion, Lichty's book is the most comprehensive and concise text available at present.   Peter Kyropoulos, USA

895. **Givens, H. C., Speed control and synchronization of multi-unit internal combustion engine plants**, *Instruments* **23**, 12, 1244–1247, Dec. 1950.

Heat and Mass Transfer

(See also Revs. 577, 738, 803, 864, 875, 942)

896. **Baker, H. D., and Laserson, G. L., An investigation into the importance of chemiluminescent radiation in internal combustion engines**, Gener. discuss. heat transfer Lond. Conf., Sept. 11–13, 1951, Sect. IV. London, Instn. mech. Engrs.; New York, Amer. Soc. mech. Engrs., 7 pp.

The distinction between chemiluminescent and thermal radiation is explained. Theoretical arguments are given, and previous experimental evidence is cited, showing that chemiluminescent radiation may play an important role in the heat-transfer processes in an internal-combustion engine and other flames and explosions. A procedure is described for determining the relative magnitudes of thermal and chemiluminescent components of the total radiation. Data thus obtained are given, indicating that the chemiluminescent component is only a small fraction of the heat transferred.   From authors' summary

897. **Beatty, K. O., Jr., Finch, E. B., and Schoenborn, E. M., Heat transfer from humid air to metal under frosting conditions**, *Engng. School Bull. N. Carolina St. Coll. Bull.* 24, 6 pp., Aug. 1951 = Gener. discuss. heat transfer Lond. Conf., Sept. 11–13, 1951, Sect. I. London, Instn. mech. Engrs.; New York, Amer. Soc. mech. Engrs.

Results are presented of heat-transfer measurements on a 4-ft, 4-in. long cylindrical surface placed in cylindrical conduits. The effects of air velocity, humidity, and test surface temperature are investigated, and the results obtained are analyzed and compared with values predicted from semi-empirical relations. Reviewer believes no definite conclusions can be drawn from present results and further investigations (experimental and analytical) are badly needed.   Y. S. Touloukian, USA

898. **Boelter, L. M. K., Young, G., Greenfield, M. L., Sanders, V. D., and Morgan, M., An investigation of aircraft heaters. XXXVII—Experimental determination of thermal and hydrodynamical behavior of air flowing along a flat plate containing turbulence promoters**, *N.A.C.A. TN* 2517, 26 pp., Oct. 1951.

This report contains heat-transfer and pressure-drop data for a system in which air flows over a heated flat plate containing

strips of metal placed normal to the direction of air flow. These strips are referred to as "boundary-layer interrupters" or "turbulence promoters."

The heat-transfer rates and static-pressure drops are increased for this system over that for a flat plate alone because of the eddies, turbulence, and fin effect caused by the interrupters. The increase of the heat-transfer rates is large when compared at equal weight rates of air. However, when compared at equal values of power consumed in pumping the air along the test section, the values of the unit thermal conductance are the same for flow over a flat plate alone, over a flat plate with either $1/_4$ or $1/_8$-in. interrupter strips, or with wooden "pin fins."

From authors' summary

899. Drake, R. M., Jr., and Kane, E. D., Heat transfer problems in high speed flows in rarefied gases, Gen. discuss. heat transfer Lond. Conf., Sept. 11–13, 1951, Sect. II. London, Instn. mech. Engrs.; New York, Amer. Soc. mech. Engrs., 5 pp.

Authors discuss approximate analyses for convective heat transfer from flat plates and spheres (using differential equations of continuous flow with transition flow effects in boundary conditions) and give experimental data for spheres when gas flow is intermediate between condition of continuum, $N_M < 0.01$ $(N_R)^{1/2}$ and free molecule flow, $N_M > 10 N_{Re}$.

M. J. Goglia, USA

900. Granovskiĭ, V. L., On steady evaporation of a fluid at different temperatures of evaporator and condenser (in Russian), Zh. tekh. Fiz. 21, 9, 1008–1013, Sept. 1951.

Author considers problem of evaporation of a liquid contained in a vessel whose walls are kept at lower temperature than surface of liquid. Boundary conditions are stated with regard to partial reflection of molecules and to deviation from Maxwellian distribution of molecular speed. In contrast to this, the treatment of the aerodynamics of vapor flow between liquid and condenser walls appears to reviewer as somewhat crude, no account being taken of possibility of more complex flow patterns due to free convection.

A. von Baranoff, France

901. Seban, R. A., and Shimazaki, T. T., Heat transfer to a fluid flowing turbulently in a smooth pipe with walls at constant temperature, Trans. ASME 73, 6, 803–807, Aug. 1951.

The case of heat transfer to an incompressible fluid of constant properties flowing turbulently within a pipe having constant wall temperature has been investigated analytically and compared to the values obtained by Martinelli for the same flow conditions in a pipe having a linear variation of wall temperature obtained from a constant heat-transfer rate along the pipe length. At low values of Prandtl modulus the ratio between the two cases may become as low as 0.73, the constant wall-temperature case being the smaller.

From authors' summary by Myron Tribus, USA

902. Berman, R., Simon, F. E., and Wilks, J., Thermal conductivity of dielectric crystals: the "Umklapp" process, Nature 168, 4268, 277–280, Aug. 1951.

The thermal conductivities of pure dielectric crystals follow a common pattern. At sufficiently high temperatures they vary as $1/T$; then at lower temperatures the conductivities increase more rapidly until tehy are limited by size effect and fall off towards zero. The shape of the conductivity/temperature curve is, to a fair approximation, determined only by θ, the specific heat parameter. It is, of course, true that the values of the conductivity in the U-region (Umklapp process region) also depend on the constant A, but this lies within relatively narrow limits.

Thus, while the magnitudes of the thermal conductivities of the different crystals vary greatly (the lowest value measured was 2×10^{-3} watt units for helium at 2 K and the highest 60 watt units for sapphire at 50 K), they can all be deduced, at least to an order of magnitude, once the specific heat parameter is known.

From authors' summary

903. Yoder, R. J., and Dodge, B. F., Heat-transfer coefficients of boiling freon-12, Gener. discuss. heat transfer Lond. Conf., Sept. 11–13, 1951, Sect. I. London, Inst. mech. Engrs.; New York, Amer. Soc. mech. Engrs., 5 pp.

904. Sharma, S. S., Thermal expansion of crystals. Part VIII. Galena and pyrite, Proc. Indian Acad. Sci. (A), 34, 2, 72–76, Aug. 1951.

Results of study of thermal expansion for galena and pyrite in the range 0–400 C are reported. It is found that the coefficients of linear expansion are given, respectively, by the formulas $a_t = 0.0_41843 + 0.0_86071\,t + 0.0_{10}1176\,t^2$, and $a_t = 0.0_58401 + 0.0_71125\,t + 0.0_{11}9168\,t^2$.

Grüneisen's constant has been evaluated and is found to vary with temperature. From author's summary

905. Allcut, E. A., An analysis of heat transfer through thermal insulating materials, Gener. discuss. heat transfer Lond. Conf., Sept. 11–13, 1951, Sect. III. London, Instn. mech. Engrs.; New York, Amer. Soc. mech. Engrs., 4 pp.

Variation of "apparent" thermal conductivity of various insulating materials, mainly of cellular type, with density and air pressure is determined experimentally. The results point out the increased relative importance of convection and radiation in heat transfer through low-density cellular materials.

John A. Lewis, USA

906. Vernotte, P., How to select the diameter in measurements of thermal conductivity by the bar method (in French), C. R. Acad. Sci. Paris 233, 4, 294–296, July 1951.

Assuming no radial variation in temperature, the maximum accuracy in determining the thermal conductivity k from a long bar heated at one end is obtained when the quantity $(L\,sh/kR)$ equals unity, where L is the distance between temperature stations, R radius, h surface coefficient of heat transfer, S cross-sectional area of the rod. Thus a small k requires a small L. For a short bar the above quantity should equal 1.5.

To keep the effect of radial temperature gradient small, R must increase as k decreases. For $k = 0.01$ cal/sec $\times$ cm $\times$ °C, L should equal 5 cm and a reasonable R equal 1 cm. For $k = 0.001$, L equals 3.5 cm and R equals 5 cm. C. F. Bonilla, USA

907. Bernath, L., Powell, H. N., Robison, A. G., Welty, F., and Wohl, K., The determination of the temperature of nonluminous flames by radiation in the near infrared, Gener. discuss. heat transfer Lond. Conf., Sept. 11–13, 1951, Sect. IV. London, Instn. mech. Engrs.; New York, Amer. Soc. mech. Engrs., 6 pp.

Characteristic radiation from water vapor at 1.346, 1.819, and 2.505 microns was analyzed and found to be thermal radiation under a variety of conditions. A Perkin-Elmer infrared spectrometer equipped with a lead-sulfide photoconductive cell was used to determine two ratios of intensity of flame radiation, and a linear relationship between the logarithms of these ratios by means of calibration against Na line-reversal results and adiabatic flame temperature calculations. A unique type of burner, using hydrogen-air and butane-air mixtures, was used to establish the fundamental relationship. Using the method described, the temperature distribution in lean and rich butane-air flames, be-

yond the primary burning zone, was determined. The paper represents a careful piece of work and constitutes a definite contribution to the field of flame-temperature measurement.

J. T. Agnew, USA

908. Griffiths, R., Thermostats and temperature-regulating instruments. 3rd ed., London, Charles Griffin and Co., Ltd., 1951, vii + 217 pp. 20s.

This descriptive book includes mainly thermostats based on the expansion of various gases, liquids, and solids, on the bimetallic strip principle, on electrical resistance, and on the potentiometer. Many of the devices are laboratory types. Chapters on low-temperature control and relays and valves are included. Many diagrams are used without which the detailed descriptions would be difficult reading.

The only mathematical analysis is an appendix called "Theoretical considerations of temperature control." This is explained partly in the Introduction: "...the approach to the problem of design is an empirical one, and the aim should be to replace it by one more fundamental."

Reviewer believes book suffers by the lack of more consideration of the fundamental problems in temperature control.

A. O. Flinner, USA

Acoustics

(See also Rev. 620)

909. Vogel, T., On vibrations of certain elastic systems in a sound field (in French), *Publ. sci. tech. Min. Air, France* no. 209, 79 pp., 1948.

Paper is an interesting study of the acoustical transmission coefficient for a wall or partition. In American usage ["Applied acoustics," Olson and Massa, p. 368, 1934] this term is defined as the ratio of the rate of flow of transmitted sound energy to the rate of the incident flow. It is a measure of the sound-insulating merit of a partition. For this coefficient, the author coins the optically inspired term "transparency." He derives an analytic expression for it in terms of air density, sound velocity, and particle velocity on each side of the partition. For a single rectangular plate type of partition the force acting on the surface is given in terms of air density, sound velocity, incident particle velocity, and plate velocity. Using the characteristic shape functions, the Lagrangian equations of motion of the plate in generalized coordinates are obtained. Thence, the required energy ratio is determined. The transmission coefficient so found is given in terms of an infinite series. A method is developed to approximate the coefficient by a finite inequality. Using Bessel functions, the coefficient for a circular plate is then determined. Next, the problem of oblique incidence is studied. The problem of double-walled partitions for normal incidence is treated in detail. The partition is considered as two simply supported rectangular plates resting on a rigid boundary. The two plates can communicate with each other by means of the enclosed air. The kinetic energy of this air is neglected in the analysis, but the potential energy represented by changes in volume is derived from the gas laws. There is considerable discussion of the air "coupling." Porous walls are treated briefly, as well as vibrations of finite amplitude.

The results of an extensive experimental investigation are given for single bare plates with and without stiffeners, double walls with air between, and plates with felt, asbestos, and glass wool affixed to the surfaces. Reasonably good agreement is shown between experimental and theoretical sound-level reductions which are shown in a series of curves plotted on frequencies as abscissae.

W. H. Hoppmann, II, USA

910. LeBel, C. J., and Dunbar, J. T., Ultra speed recording for acoustical measurement, *J. acoust. Soc. Amer.* 23, 5, 559–563, Sept. 1951.

Paper deals with recording of rapidly fluctuating voltages on a logarithmic scale, e.g., for reverberation measurements. Logarithmic conversion is done by means of Varistor systems, which are not described in detail; recording is done with a direct writing oscillograph, fed by a push-pull d-c amplifier. Writing speeds up to 10,000 db/s are attained, compared with up to 500 db/s with conventional high-speed level recorders, as stated by the authors. See, however, P. V. Brüel and U. Ingard [J. acoust. Soc. Amer. 21, 91–93, 1949]: 1000 db/s. Possible application: reverberation-time measurements in rather dead, small studios.

G. J. van Os, Holland

911. Kaspar'yants, A. A., On the propagation of sound from a flat pulsating radiator (in Russian), *Prikl. Mat. Mekh.* 15, 4, 445–450, July–Aug. 1951.

Author considers the decay of the velocity potential of a field of sound generated by a harmonically vibrating piston in a flat infinite wall, after the piston has suddenly stopped moving.

Part 2 deals with the establishment of the field during the transient state, when the piston suddenly begins vibrating. In both cases, the velocity potential in a point M at time t after the piston has stopped or begun, is given by an integral over the part of the piston lying within the sphere with radius ct and center M.

W. H. Muller, Holland

912. Kenworthy, R. W., and Burnam, T. D., The absorption coefficients of fir plywood panels, *J. acoust. Soc. Amer.* 23, 5, 531–532, Sept. 1951.

The absorption coefficients of flat, splayed, and cylindrical panel assemblies made from one-quarter-inch three-ply fir plywood have been measured by the reverberation-chamber method under three different conditions. All panels have greater absorption at the lower frequencies (128 and 256) than at higher frequencies. The effective absorption of each type can be increased by using a half-inch kapok felt in combination with it, either mounted on the wall behind the panel or spot-cemented to the plywood. Each type of panel has desirable characteristics under certain conditions. The absorption of all types indicates that such panels are important acoustical materials in the lower frequency range.

From authors' summary

913. Nimura, T., The diffraction of sound by circular apertures, *Sci. Rep. Res. Inst. Tôhoku Univ.* (B), 1—2, 3, 381–389, Mar. 1951.

Author has developed a theory for the diffraction of sound by a circular aperture using spheroidal wave functions. Acoustic impedance, directional characteristics, and transmission of acoustic output through the aperture have been determined in agreement with Strutt's results for $2\pi(r/\lambda)$ and $f = 870$ cycles. Results are interesting in the sense that they form an extension of Lamb's results, where the radius is very small compared to the wave length, to the general case where the ratio r/λ may be large or small. Its technical importance may be enhanced if the results are found to hold good in the ultrasonic range, which needs investigation.

Gurdeva Sharan Verma, India

914. Levitas, A., and Lax, M., Scattering and absorption by an acoustic strip, *J. acoust. Soc. Amer.* 23, 3, 316–322, May 1951.

Authors give three reasons for reconsidering a problem that has already been partly solved by Pellam [title source, 11, p. 396, 1940]: (1) The methods applied here are more powerful and can be generalized more easily. Ellipsoidal coordinates cannot be

used at all for other shape strips. (2) It will be possible to test the accuracy of the variational procedure in a problem involving absorption by comparing with Pellam's result for the absorption cross section. (3) Pellam did not calculate the scattering cross section of the strips and this is as important to room acoustics as the absorption cross section.

Authors find an integral equation for the pressure more convenient than the differential equation. They prefer this type of equation as they are more interested in the scattering amplitude in various directions than in the pressure on the strip. By means of the integral equation, a variational expression is constructed for the scattering amplitude in terms of the pressure on the strip. They claim that this procedure has two advantages: (1) There is no need to solve integral equations; (2) explicit expressions are obtained in terms of the physical quantities, such as frequency strip width, impedances, etc., instead of numerical results.

Paper ends with the following conclusions: (1) The combination of Green's function, integral equation, variational methods and cross-section theorem forms a powerful technique in the solution of scattering problems; (2) comparison with Pellam's "exact" numerical calculation indicates that the technique used has an accuracy better than 2%; (3) the narrowest strips are most effective; (4) maximum scattering of sound of wave-length λ occurs with strip $2\lambda/3$ wide.

Salih Murat Uzdilek, Turkey

915. Chester, W., The propagation of a sound pulse in the presence of a semi-infinite open-ended channel. I, *Philos. Trans. roy. Soc. Lond.* (A) 242, 854, 527–556, Sept. 1950.

Paper treats the behavior of a sound pulse propagating and diffracting in the neighborhood of the open end of a semi-infinite channel of constant width. Two problems are considered. The first corresponds to the sound pulse originating inside the channel, the second to the sound pulse originating outside. It is classical in nature. Similar problems have been treated by Summerfield (1894, 1901) and Levine and Schwinger [AMR 3, Rev. 986]. Following the recent development on supersonic wing theory of Gunn [see AMR 2, Rev. 898], author attacks the problem with operational calculus and obtains an appropriate Green's function. A reciprocity relation is found so that the solution of the second problem can be derived from the first.

Chieh-Chien Chang, USA

916. Rust, H. H., Generation of ultrasonics by means of volume-magnetostriction (in German), *Z. angew. Phys.* 3, 1, 9–14, 1951.

A method of developing ultrasonic vibrations by means of magnetostriction is described. A mixture of metallic powder dispersed in an electrically insulating liquid is subjected to a rapidly changing field, thereby causing volumetric changes due to magnetostriction. The resulting waves of ultrasonic frequency can be used as driving sources. Fortunately, some of the best materials are relatively cheap metals such as "carbonyl iron powder EN."

A. G. H. Dietz, USA

917. Gopalji, Ultrasonic absorption in normal air at 456 kc/s for different humidities, *Indian J. Phys.* 25, 6, 298–304, June 1951.

Author gives purported "simpler treatment" of method used to get absorption constant of sound wave in interferometer, then reports measured results at 455.8 kc in air of relative humidities between 31 and 70% (5 points). He finds maximum absorption near 48% R.H. Theoretical treatment is "simpler" only by original neglect of effects shown negligible by others. Author's assumed form for attenuated plane wave is, unfortunately, *not*

attenuated as it travels, owing to cancellation of space and time parts. However, experimental results do not suffer, for they are calculated with standard final equation.

A. O. Williams, Jr., USA

918. Hart, R. W., Sound scattering of a plane wave from a nonabsorbing sphere, *J. acoust. Soc. Amer.* 23, 3, 323–329, May 1951.

By a method applied previously to the corresponding electromagnetic problem, approximate closed form analytic expressions for the reflectivity (angular variation of magnitude of asymptotic pressure in scattered wave) and total cross section σ_t are found for spheres of arbitrary size. The scattering depends on two parameters, the density ratio μ and sound velocity ratio m, of material in sphere to surrounding medium. The approximation is good when $m - 1$ and $\mu - 1$ are small. Comparison of exact results with approximate σ_t over a range of sphere radius a shows: (1) for a large and (2) for a small (Rayleigh limit), compared to free-space wave length λ_0, errors of a factor of magnitude (μm)?, (3) for $m = 5/4$, $\mu = 1$ and $m = 5/6$, $\mu = 1$ good agreement, (4) for $m = 1$, $\mu = 2$, only qualitative agreement. Comparison of reflectivity in case (3) shows good agreement in forward direction for $k_0 a = 2\pi a/\lambda_0 = 4$, but poor for large scattering angles (where little scattering occurs) although agreement improves for smaller $k_0 a$.

The mathematical procedure sums the rigorous Rayleigh expansion of the scattered wave pressure in spherical wave functions by approximating the denominators of the coefficients. The asymptotic forms of the Bessel functions are used valid for $(k_0 a, m k_0 a) >> n$, the order of the Bessel functions in the nth term, but approaching validity for any n for $\mu - 1$, $m - 1$ small; n then drops out of the denominators and the series for the asymptotic scattered wave is summed exactly by an addition theorem. Closed form integration to obtain σ_t requires neglecting some terms of order $\mu m - 1$ compared to those retained.

Paul Marcus, USA

919. Petralia, S., and Cevolani, M., Ultrasonic velocity in supercooled liquids (in Italian), *Ric. sci.* 21, 9, 1623–1625, Sept. 1951.

Author describes measurements of the velocity of ultrasonic oscillations made in normal and supercooled phenylsalicylate. No sudden variation of velocity was observed at passage of temperature through melting point; moreover, velocity results are independent from frequency. Values are finally given of adiabatic and isothermal compressibility of phenylsalicylate at 44 C.

From authors' summary

920. Anderson, N. S., and Delsasso, L. P., The propagation of sound in carbon dioxide near the critical point, *J. acoust. Soc. Amer.* 23, 4, 423–429, July 1951.

Experimental and theoretical values for sound velocity and experimental values for sound absorption are presented in the form of isotherms near the critical-point temperature plotted against pressure or specific volume. The theoretical values are based upon a simple velocity expression derived from the isentropic partial derivative of pressure with density by means of elementary calculus and thermodynamics. This expression is evaluated using incrementally calculated values for its partial derivatives using van der Waals equation of state and the Beattie-Bridgeman equation of state. Experimental work is done with a Pierce-type variable path acoustic interferometer within a pressure tank, using 572 kc sound. Velocity differs between liquid and vapor below critical point, is 142 m/sec (experimental) at critical, falls with rising pressure above critical through a sharp minimum into

an abrupt rise. Attenuation decreases with rising pressure until critical point is reached, where it rises abruptly. Agreement between theory and experiment is satisfactory.

Robert B. Green, USA

Soil Mechanics, Seepage

(See also Rev. 586)

921. Pottier, F., supplemented by **Chalos, M.,** and **Vignal, J.,** Earth transportation on a horizontal two-dimensional surface (in French), *Ann. Ponts Chauss.* **121,** 3, 249–283, May–June 1951.

A new method is presented for finding the most inexpensive distribution of earth, excavated over a great area (airfields, etc.). Author solves the problem for a restricted case by means of very simple principles (principles of continuity, Monge, unclosed polygons, quadrangle, etc.). At end, author shows a practical application.

Reviewer believes the results are of little use for engineers.

H. H. Ravn, Sweden

922. Vyalov, S. S., Limit equilibrium of weak soils on a rigid base (in Russian), *Izv. Akad. Nauk SSSR Otd. tekh. Nauk* no. 6, 813–828, June 1951.

Problem of limit equilibrium for half plane, loaded at a part of the boundary with uniformly distributed load p, was formulated and solved by L. Prandtl [*ZAMM* **1,** 1, 1921], who formulated also [ibid., 3, 6, 1923] the analogous problem for a long strip on a rigid base, similarly loaded. Solution of this latter problem, under Saint Venant's condition of plasticity, was elaborated by Sokolovsky ["Theory of plasticity," Moscow, 1946, pp. 163–165] by means of Massau's numerical method of integration. Present paper deals with the same problem, but under Prandtl's (more general) condition of plasticity $\tau = A - B\sigma$. Solution is also obtained by Massau's method. The influence of side parts of the strip (which are not plastically deformed) on the critical value of the load p is approximately evaluated because the exact solution would involve the mixed elastic-plastic problem. Detailed numerical solution of an example is given and conclusions are drawn concerning the estimation of the bearing capacity of foundations. Dragoš Radenkovic, Yugoslavia

923. Uchida, S., On the theory of percolation with surface of seepage (in Japanese), *Rep. Inst. Sci. Technol. Tokyo* **4,** 7/8, 200–204, 1950.

In view of the fact that the seepage of rain water has a large influence upon the character of percolation flow in the dam, author presents a method for solving the equation for the potential of percolation flow with seepage surfaces using conformal transformation. At the seepage surface, either pressure or velocity of penetration may be assumed to be constant; selection between the two assumptions seems to depend upon the conditions, such as quantity of rain water and others. Several examples of exact solution for trapezoidal sections are presented. Agreement with the results of sand experiment is satisfactory.

Humio Tamaki, Japan

924. Uchida, S., On percolation in earth dams (Observation in Yamaguchi dam). **I & II** (in Japanese), *Rep. Inst. Sci. Technol. Tokyo* **4,** 1/2, 3/4; 39–44, 92–97, 1950.

Part I deals with the mathematical formulation of the percolation problem and describes the properties of the soil constituting the Yamaguchi dam. Effective permeabilities of various parts of the dam are estimated from the composition of the soil. Negative pressure due to capillarity is also estimated.

Part II analyzes the observation. Equipotential and streamlines are determined from the measurements of water pressure with 21-gage tubes. Seepage of rain water through slanting surface is found to play an important part in the flow pattern. Taking this effect into account, author gives a numerical solution by means of difference method under simplified boundary conditions. Calculated flow pattern is nearly the same as the observed. Humio Tamaki, Japan

925. Shchelkachev, V. N., Investigation of the unsteady filtration flow of an elastic liquid in a circular battery of drains (in Russian), *Dokladi Akad. Nauk SSSR (N.S.)* **79,** 4, 577–580, Aug. 1951.

To study problems of oil production, a theoretical analysis is made of the pressure-time histories at all points in a two-dimensional infinite oil field containing a circular array of equally spaced wells drawing oil at a constant rate. Viscosity and elasticity of the oil are taken into account. The analysis is based on a well radius which is negligible compared with the radius of the circle on which the wells are located. The solution is extended to the case of a continuous annular producing ring replacing the circular array of wells. Walter W. Soroka, USA

926. Angervo, J. M., Snow investigation in Finland (in Swedish), *Tekn. Tidskr.* **81,** 34, 747–749, Sept. 1951.

Various methods for studying snow formations and their characteristics have been developed in the past. Paper deals with a Finnish program of investigating sheets of snow covering the ground. It has been found suitable to define different kinds of snow by certain mechanical properties such as particle size, porosity, and wetness. Density and hardness are determined by an apparatus specially devised for the purpose. Aside from its physical, hydrological, and climatological interest, problem may be of importance in the design of tools and machinery for handling snow on roads, etc. Method was applied with advantage in the field during winters of 1948–49 and 1949–50, for which the characteristics of the snow cover in various parts of Finland are shown. P. Wilh. Werner, Sweden

927. Ruppeneit, K. V., Checking of strength of blocks of given dimensions (in Russian), *Izv. Akad. Nauk SSSR Otd. tekh. Nauk* no. 7, 1065–1084, July 1951.

Paper presents a continuation of the previous investigation [AMR 4, Rev. 2753] of the carrying capacity and the state of stress in the blocks of mineral substance left unexcavated in mines for the support of the roof of underground chambers. The material in these blocks is assumed to be in the state of limiting equilibrium characterized by the shape of the envelope to the Mohr's circles of stress at failure, consisting of a cycloid passing into an inclined tangent straight line. Three parameters describing the strength properties of the material and susceptible to experimental determination are necessary to conform to the assumed shape of the envelope. The blocks are considered in the form of solid walls left between parallel horizontal passages. Determination of stresses and directions of the surfaces of sliding in different regions of the block is made partly by explicit formulas and partly by means of a step-by-step solution of differential equations. In some cases the straight line part of the envelope is effective alone throughout the whole block. Different mechanical conditions with regard to the shearing strength at top and bottom contact surfaces of blocks are considered, and their effects on stresses in the blocks are evaluated. The contact shearing strength may be the same as elsewhere in the block, may consist of friction alone, may consist of both friction

and cohesion of different intensities than in the block itself, or may be zero.

Several numerical examples are solved. The possibility of progressive failure is not visualized.

Alexander Hrennikoff, Canada

928. Hauth, W. E., Jr., and Davidson, D. T., Studies of the clay fraction in engineering soils, *Nat. Res. Counc. Highway Res. Board; Proc. 30th ann. Meeting,* 449–464, 1951.

Studies of clay particles found in soils used for structural purposes are reported. Seven different soils were investigated. Techniques and results are presented for identification of clay mineral type, evaluation of particle-size distributions to diameters of minus one micron, and rapid determination of soil cation-exchange capacity. Differential thermal analysis is used for mineral identification. The soil sample and an inert standard are slowly heated in a furnace. Characteristic thermal patterns of temperature differences result from the various sample reactions (e.g., crystallization, inversion, loss of water, crystal structure breakdown, etc.). With certain samples, preliminary preparation procedures are necessary to concentrate the clay fraction or eliminate interferring constituents. The particle-size determination is a modified soil-hydrometer procedure using the long arm centrifuge. The technique used to measure the cation-exchange capacity is adopted from the method of I. C. Brown. The pH of the sample is taken after treatment with normal ammonium acetate to evaluate exchangeable hydrogen, and after treatment with normal acetic acid for exchangeable bases.

Reviewer agrees with author's suggestion that techniques constitute valuable addition to our understanding of the role of clay fraction in engineering soils. Before correlations of results obtained with these methods can be made to soil properties, further advancement of fundamental soil science is essential.

Thomas L. Speer, USA

929. Watanabe, T., Rapid and easy method for determining the degree of compactness of soil by electrical resistivity (in Japanese), *J. Soc. civ. Engrs. Japan,* 13 pp., May 1951.

After performing some careful experiments on the electrical resistivity of soil in the laboratory, the applicability of the method (analogous to that of the electrical resistivity exploration) to the field determination of the relative compactness of soil is suggested. Author ascertains that it affords a more rapid, less laborious, and more exact method than the ordinary one.

T. Mogami, Japan

930. Haefeli, R., Compressibility of soils (in French), *Minist. Obras Públ., Labor. Engen. Civil, Lisboa Publ.* 16, 13 pp., 1951.

In computing soil settlements, writer uses the elasticity module, determined from oedometer tests. This elasticity module has no constant value but varies with the consolidation pressure of the soil. As in sampling operations, the soil is always unloaded and often remolded to a certain extent; the writer completes and controls the results of oedometer tests with direct loading tests on the soil in situ. To that end, a light and a heavy movable apparatus are used with loading plates of 100 and 1000 cm², respectively.

On the Kloten airfield the results of these tests show the close relation between the elasticity module and the pressure used in these tests.

F. C. de Nie, Holland

931. Lambe, T. W., Capillary phenomena in cohesionless soils, *Amer. Soc. civ. Engrs. Proc.* Separate no. 4, Feb. 1950 = *Trans. Amer. Soc. civ. Engrs.* 116, 401–423, 1951.

A fine sand was tested in tubes for horizontal and vertical capillary flow of water including drainage from saturated and partly saturated conditions. The water pressures at various times along the tube and the final moisture distributions are presented and compared with theoretical results for two interconnected tubes of different diameter. Characteristic capillary heads are indicated and discussed. Further research is suggested, including the development of standard drainage testing equipment.

Edward S. Barber, USA

932. Dykstra, H., and Parson, R. L., Relaxation methods applied to oil field research, *J. Petr. Technol. Trans.* 3 (192), 8, 227–232, Aug. 1951.

Illustrative examples are presented for two- and three-dimensional steady flow. Applications to core analysis and to the case of nonisotropic permeability are outlined. Computed pressure drops are compared with those measured in a Hassler type relative permeability apparatus.

Donald M. Vestal, Jr., USA

933. Topping, A. D., Time-dependent strains around uncased holes, *World Oil* 133, 1, 134–138, July 1951.

Approximate solution for viscoelastic flow around a vertical cylindrical hole in a semi-infinite gravitating body is applied to problem of gradual closing of bore hole. Observed rates of closing can be explained only if viscosity of rock is smaller than any values yet measured.

F. R. N. Nabarro, England

Micromeritics

934. Davies, C. N., and Aylward, Mary, The trajectories of heavy, solid particles in a two-dimensional jet of ideal fluid impinging normally upon a plate, *Proc. phys. Soc. Lond.* (B) 64, part 10, 382B, 889–911, Oct. 1951.

Solution is given for frictionless incompressible flow of a plane, free jet leaving a parallel-sided channel and impinging on an infinite perpendicular plate. Then the trajectories of small, solid, spherical particles initially carried with the uniform channel flow are computed by a stepwise procedure, using the Stokes (small Reynolds number) approximation for the relative motion. Principal results are minimum values of dimensionless particle parameter $(P = mU_0/3\pi r\eta h)$ for occurrence of impaction, as function of original particle-path location across the channel and of relative distance from channel exit to plate; m = mass, U_0 = velocity far upstream in channel, r = radius, η = fluid viscosity, h = channel width. Authors mention possibility that, in practice, jet turbulence and channel and plate boundary layers may have some effect, but make no effort to justify the neglecting of these phenomena.

Stanley Corrsin, USA

935. Trevelyan, B. J., and Mason, S. G., Particle motions in sheared suspensions. I. Rotations, *J. Colloid Sci.* 6, 4, 354–367, Aug. 1951.

The rotational velocity of single glass spheres (diam 150 and 290 μ) was measured at rates of shear from 0.068 to 1.57 sec^{-1}. A special Couette apparatus in which two coaxial cylinders rotated in opposite directions was used. The experimental periods of rotation were in excellent agreement with the theoretical values as predicted by Jeffery's equations.

The periodic motion of cylinders followed Jeffery's equations for prolate spheroids except that the calculated axis ratio is significantly less than the measured value. The cylinders were made from bundles of glass fibers embedded in Tissuemat.

H. E. Robison, USA

Geophysics, Meteorology, Oceanography

(See also Revs. 590, 747)

936. Platrier, C., Contribution to Foucault's theory of pendulum (in French), *Bull. Acad. roy. Belg. Cl. Sci.* (5) 37, 8–9, 762–779, 1951.

Author derives the equations of motion of Foucault's pendulum, using an approximation valid for small vibrations, and gives a solution by means of elliptic functions. In particular, he investigates the horizontal projection of the spherical trajectory of the particle. He adds some graphs of the different types of this curve.   O. Bottema, Holland

937. Schmidt, F. H., Streamline patterns in equatorial regions, *J. Meteor.* 8, 5, 300–306, Oct. 1951.

Author derives expressions for wind streamlines in the equatorial regions based on certain simplifying assumptions. Since geostrophic components are not important in these regions, Rossry's vorticity equation is used as the basis for the derivations with assumptions of horizontal motion alone and the dependent variables as being functions only of displacement along the meridian.

The particular cases treated in paper are: Monsoon motion across the equator, doldrums and high pressure areas on the equator, and an east-west line of convergence displaced from the equator. In general, the derived streamlines are in qualitative agreement with those actually observed.

Warren W. Berning, USA

938. Imahori, H., and Hori, J., On the diffusion by turbulent motion, *J. meteor. Soc. Japan* (2) 29, 10, 327–335, Oct. 1951.

The intention of the authors is to clarify the analysis of turbulent diffusion in the atmosphere, taking into account the fact that averages can never be made over times large compared with the period of the slowest fluctuations. In reviewer's opinion, no clarification is achieved and most of the assumptions suggested seem very dubious. The sections on the analysis of two-dimensional Markoff processes by means of the Fokker-Planck equation are clear and interesting but of doubtful relevance.

G. K. Batchelor, England

939. Ichiye, T., On the effect of lateral mixing in the ocean current, *Oceanogr. Mag. Centr. Meteor. Observ. Tokyo* 2, 3, 105–111, Sept. 1950.

Published independently at about the same time as Munk's paper [see AMR 4, Rev. 3094], a solution is obtained for the mass transport function of ocean circulation due to an idealized wind, in very good agreement with that of Munk.

Henry Stommel, USA

940. Saito, Y., On the problem of the stationary drift current in an anisotropic ocean (in Japanese), *J. meteor. Soc. Japan* (2) 28, 35–61, 1950.

Author attempts to obtain a complete analytical solution for the problem of the stationary drift current in the ocean which is bounded by two parallel coastal lines. Solving equations of motion of anisotropic turbulent fluid on a rotating earth by using Fourier integral theorem and Stokes's method, he obtains an analytical solution but does not give any numerical discussion.

S. Syōno, Japan

941. Kuo, H., Vorticity transfer as related to the development of the general circulation, *J. Meteor.* 8, 5, 307–315, Oct. 1951.

Author discusses formation and maintenance of zonal wind systems by investigating the vorticity transfer by atmospheric vortexes. Equation for the zonal angular momentum is integrated along latitude circles and over the height of the atmosphere under the assumption of zero mean flow across latitude circles. Introduction of vorticity terms into the resulting equation shows that the change of mean zonal momentum is equal to the transport of vorticity minus frictional losses. Vorticity transfer is mainly due to large-scale eddy motions. This eddy transfer is discussed by using the two-dimensional vorticity equation, neglecting all terms but those for horizontal advection. A tendency results for any disturbances with concentration of vorticity to transfer vorticity, northward under normal atmospheric conditions. They tend to approach regions of same absolute vorticity as their own. Inside a zonal belt with active disturbances, gradient of absolute vorticity is thereby increasing, and zonal wind fields can result. The time necessary for such a development is calculated to be three weeks, and mean seasonal currents can be derived in agreement with observations. Paper is based on three previous ones of same author; see AMR 3, 4; Revs. 2127, 1402.   Horst Merbt, Sweden

942. Kannuluik, W. G., and Carman, E. H., The temperature dependence of the thermal conductivity of air, *Austral. J. sci. Res.* (A) 4, 3, 305–314, Sept. 1951.

The thermal conductivity of dry air has been determined at five standard and substandard temperatures (boiling point of oxygen: $-183°$; sublimating point of carbon dioxide: $-78.5°$; melting point of ice; boiling point of water; and boiling point of naphthalene: 218 C) by the "thick wire" variant of the "hot wire" method. The air pressures ranged from 0.5 to 76 cm of mercury.

The method, as used earlier in measurements at 0 C by Kannuluik and others, entailed the observation of the rate of heat transfer through the air from an electrically heated vertical platinum wire (11.6 cm long, 1.5 mm in diam) to a coaxial platinum-iridium cylinder (7-mm internal diam), maintained at any one of the above temperatures. Convection currents were entirely absent. A small radiation correction (as determined by making observations on the evacuated apparatus) was applied at the higher temperatures. Due to the relative thickness of the hot wire, no correction for temperature discontinuity at the surface of the wire needed to be applied except at the highest temperature and for the lower pressures. A small correction (not explained in detail in this paper) of less than 1% for the deviation from radial flow was also found necessary.

The results confirm the independence of the thermal conductivity of the pressure in the pressure range 6 cm mercury to atmospheric. The temperature dependence can be expressed by the relation

$$k = 5.75 \times 10^{-5} (1 + 3.17 \times 10^{-3}t - 2.1 \times 10^{-6}t^2)$$

(with t in degrees centigrade and k in cal deg^{-1} sec^{-1} cm^{-1}). At 0 C, the value 5.75×10^{-5} for k is in fair agreement with earlier determinations by the same method as well as values obtained by the "parallel plate" method.

Walter Hitschfeld, Canada

Lubrication; Bearings; Wear

(See also Rev. 676)

943. Blok, H., Fundamental mechanical aspects of thin-film lubrication, *Annals N. Y. Acad. Sci.* 53, 779–804, June 1951.

Thin-film lubrication lies between full hydrodynamic and boundary lubrication. Author contends that "oiliness," an

important surface-active property in thin-film lubrication, cannot be isolated by eliminating the influence of viscosity alone. Condition of rubbing surfaces, viscosity and temperature variations in the film, and elastic deflections of the structural members must also be accounted for.

Author presents correlational data from literature and his own extensions for each of the above effects separately, concluding with a general discussion of combined effects. Although the information is qualitative and not directly usable for design purposes, reviewer believes that the ideas form a good basis for further research and quantitative correlation.

Eugene M. Simons, USA

944. Cheyney, L. E., Mueller, W. J., and Duval, R. E., Frictional characteristics of O-rings with a typical hydraulic fluid, *Trans. ASME* 72, 3, 291–297, Apr. 1950.

O-ring packings employed in aircraft hydraulic systems are subject to numerous operating variables, which affect the friction involved in their operation. A method for studying the friction is described, and the effects of several variables—pressure, time delay, squeeze, stroke speed, ring size, and surface finish of the moving metal part—have been determined.

From authors' summary

Marine Engineering Problems

(See also Revs. 687, 861)

945. de Dinechin, G., Note on the slip of a propeller working under various conditions (in French), *Bull. Assn. tech. marit. aéro.* no. 49, 139–148, 1950.

Assuming that the resistance depends on the square of speed of advance and, further, that both thrust deduction and wake are independent of speed, a relation is deduced between the advance coefficients of the propeller when varying the resistance

of the same ship, e.g., submarine on surface and submerged.

H. W. Lerbs, USA

946. van Lammeren, W. P. A., Design problems of screw-propellers (in Dutch), *Voordr. Konink. Inst. Ing. (Oosthoek, Utrecht)* no. 47 t/m 58, 933–960, Sept. 1951.

Paper deals with requirements with regard to designing, making, and finishing of ships' propellers. Value of application of standard propeller charts for design of propellers is underlined. An extension is given of the systematic propeller series diagrams of the Wageningen Ship Model Basin for blade-thickness ratios which differ from standard thickness ratio of 4-bladed B-series propellers. Effect of accuracy of make and finish of full-scale propellers upon their efficiency and cavitation properties is discussed briefly. Finally, a standardization sheet for allowable deviations from dimensions of screw propellers is given and elucidated in detail.

From author's summary by Georg Vedeler, Norway

947. Allan, J. F., and Conn, J. F. C., Effect of laminar flow on ship models, *Shipbuilder* 57, 498, 270–274, Apr. 1950.
See AMR 5, Rev. 319.

948. Mathews, S. T., Resistance and propulsion tests on a model of a lake freighter, *Nat. Res. Counc. Canad. mech. Engng. Rep.* MB-137, 12 pp., 10 figs., July 1951.

An account is given of resistance and propulsion tests, with three alternative screws, on a lake freighter model. Resistance test results are given for three conditions over a speed range of 7.5 to 12.5 knots. Propulsion results are presented from 10.5 to 12.5 knots for the design water line. There is an analysis of the results, together with a general discussion of their significance.

From author's summary

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Martin Goland, *Editor*

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